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Location choice when the number of jobs matters.

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Abstract

The idea that people want to go to where the jobs are is intuitive, yet is absent from the standard quantitative spatial modelling approach in which location choices are guided by prices, without reference to quantities (the number of jobs in a place). The purpose of this paper is to fill this gap by making jobs, as well as places, the objects of household choice. This involves minor change to the modelling approach used in the literature and provides a simple description of labour market matching. Similar modification of the modelling of firms' location choices captures the idea that these are shaped by both wage costs and the availability of workers with appropriate skill. These modifications yield powerful agglomeration forces, as workers' location choices become positively influenced by the number of jobs in a place, and firms' decision are shaped by the number of workers of appropriate skill levels. Results are established analytically and in a regional model in which the equilibrium distributions of workers and sectors are demonstrated.

Keywords: Spatial models, agglomeration, discrete choice, matching.

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1. Introduction:

A frequent claim in discussion of regional disparities is that workers will not move to a place because there are no jobs, and firms will not move because there is a shortage of workers, particularly those with required levels of skill. This suggests that, while wages and prices matter for location decisions, so too do quantities – the number of jobs and workers with appropriate skills in each place. The approach of much economics is however to model these choices as depending exclusively on wages and prices, as is the case in the literature on quantitative spatial models (QSM). In this literature it is assumed that workers' location choices depend on wages, the cost of living (including commuting costs) and intrinsic locational preferences, but not on the number or availability of jobs in each place. This paper shows how the approach taken in this literature can be generalised to include the number of jobs in a place in households' choice functions, as well as the number of workers in a place in the location decisions of firms. This is a powerful additional force for agglomeration, as is demonstrated in a QSM setting.

The standard urban QSM framework contains a discrete choice model in which the objects of household location choice are place of residence and place of work, the two being distinct if commuting is possible, or combined if not.[1] This ignores the fact that, in many occupations, it seems plausible that the object of choice is a job, not only, or primarily, a location in which to work. Jobs are located in places, so choice of a job induces choice of place, but the fundamental object of choice is often the job, not the place. Furthermore, places are inevitably somewhat arbitrary constructs of the model or of pre-existing administrative or statistical classifications; there is little reason to think that – particularly for places of work – they coincide with peoples' preferences. By contrast, a job is an clearly defined economic object.

Working with jobs as an object of choice, instead of or as well as places, makes a difference if households have idiosyncratic valuations of different jobs, arising from intrinsic preferences or from worker-job match-specific productivity. More jobs in a place offers the prospect of a better match, so means that the number of jobs matters for location choice. This observation is, of course, not new; matching is one of the central pillars in Duranton and Puga's 2004 handbook chapter. By thinking of jobs as a fundamental object of choice, this paper provide a simple way of capturing this, building directly on – and as a straightforward generalisation of -- the discrete choice approach taken in QSM models.[2]

Locational choices are made by households and workers, and also by firms. In the QSM approach firms are generally given a passive role, simply locating where there are workers. In this paper we apply the discrete choice apparatus to the decisions of firms, as well as those of households, on the basis that productivity may depend on both worker and place specific factors. Places with many workers are then, given other factors, relatively attractive to firms as they offer the potential for better worker-job

[1] A useful survey of the QSM literature is Redding and Rossi-Hansburg (2018). Major contributions include Ahlfeldt et al (2015), Monte et al. (2018), Heblich et al. (2020). For theoretical foundations see Thisse et al. (2021). Further applications of this location based preference modeling include Allen and Donaldson (2022) Miyauchi et al. (2022), Heblich et al. (2022), Kleinman et al. (2023).

[2] The extensive literature on matching (surveyed in (Petrongolo and Pissarides 2001) has been applied in a spatial context in papers including: Helsey and Strange (1990), Amiti and Pissarides (2005), matching with market power (Matouschek and Robert-Nicoud 2002) and with signalling (Venables 2002). We do not address sorting between workers and cities of different types, as in the literature on sorting surveyed by Behrens and Robert-Nicoud (2015).

matches. This analysis builds once again on the discrete choice model, analogous to our treatment of household choice. Outcomes are then determined by the interaction of the location decisions of firms as well as households, with equilibria where these choices coincide. The stability of an equilibrium – and hence the location of activity – depends in a transparent way on features of land and housing supply (an elasticity of the cost of living with respect to population), on technical agglomeration economies (elasticity of productivity with respect to employment), and on Frechet shape parameters, capturing the distribution of the valuations that households and firms attach to places, to jobs and to workers.

We develop these arguments in three stages. Section 2 of the paper sets out the household choice problem and shows how making jobs an object of choice creates an additional agglomeration force. Section 3 turns to firms' location decisions, and to how these decisions interact with those of households. Section 4 draws out implications for equilibrium outcomes in a model with two labour types, two sectors (using labour types in different proportions) and two regions; workers choices depend on available jobs of their own type. Depending on parameters there is a rich pattern of possible equilibria, some with full or partial sectoral specialisation of regions. Section 5 discuss extensions, including the extension from the regional to the urban, in which commuting is permitted so choices are made of job, place of work, and place of residence.

2. Household location and labour supply

The approach builds on the idea that individuals make discrete choices over objects – such as places to live or jobs to hold – and these choices are based on economic variables such as the real wage, and on idiosyncratic valuations or preferences that are distributed through a large population. It is widely assumed in QSM models that the distribution of these valuations in the population is Frechet, this supporting a simple CES-like description of aggregate choices in which the shape parameter measures the heterogeneity of valuations across the population. The smaller is this parameter the greater is the heterogeneity implying choices are, in aggregate, less sensitive to economic variables such as the wage. The approach also gives a simple formula for average household utility levels. We apply this framework first where objects of choice are places, and then when they are places and jobs in each place.

The simplest version of the standard case is that in which there is a single type of object of choice, the place or region of work (absent commuting, the same as region of residence). There are N regions, labelled $n, m \in \{1 \dots N\}$, each of which offers a real income to each household denoted v_n; we call this the base income of region *n*. There are H households, $h \in 1 \dots H$, and the utility of household *h* working in region *n* is $v_n b(h,n)$, the product of base income and a household specific valuation (or preference parameter) $b(h,n), h \in H, n \in N$. The valuation $b(h,n)$ of household *h* for work in *n* is drawn from a Frechet distribution with shape parameter $\eta > 1$ and scale parameter 1. The product $v_n b(h,n)$ is therefore Frechet with shape η and scale v_n. Each household chooses its preferred region and, using properties of the Frechet distribution, the proportion of the population choosing region n (denoted Π_n) and expected household utility u (given optimal location choice) are respectively

$$\Pi_n = v_n^\eta / \sum_{m=1}^{N} v_m^\eta, \qquad u = \Gamma\left(\frac{\eta-1}{\eta}\right)\left(\sum_{m=1}^{N} v_m^\eta\right)^{1/\eta}, \tag{1}$$

where $\Gamma(1-1/\eta)$ is the gamma function, with $\Gamma(1)=1$ and increasing with $\eta>1$.[3]

These equations contain three important properties. First, high η means low variance of valuations $b(h,n)$, so aggregate choices as captured in Π_n are more sensitive to the economic fundamentals in base income, v_n; low η means individuals' choices are strongly shaped by their intrinsic place valuations. Second, expected utility is the same for all regions (does not depend on *n*); a region with low v_n is only chosen by households with high preference factor, $b(h,n)$, these effects exactly offsetting in the average. Third, expected utility, *u*, is increasing in *N* (adding more elements to the sum in expressions (1)), and more so the lower is η. The intuition comes from the fact that each household chooses its preferred region, so this maximum increases with the number of choices available.

We now extend this to make households' objects of choice both regions and jobs, each of these with associated idiosyncratic (household specific) valuations. We do this as a two-stage nested choice problem. At the second stage households choose a job in each region, based on base income and idiosyncratic valuations over jobs, this giving an expected utility level for the region. At the first stage they choose region, based on expected utility for each region and idiosyncratic valuations over regions. We continue to work with these valuations being drawn from a Frechet distribution, i.e. constructing a nested Frechet decision process, analogous to a nested multinomial logit (see Anderson et al. 1992 and appendix 1).

Looking first at the second stage, households in each region *n* choose a particular job, drawn from the number of jobs in the region, L_n, these summing to the total number of jobs in the economy, $\sum_{n\in N} L_n = L$.[4] Jobs in region *n* are indexed $j|n$, with *j* running $j \in 1 \ldots L_n$. A particular job $j|n$ in region *n*, has base income $v_{\,j|n}$, and $b(h,j|n)$, is the valuation household *h* places on working in this job. These valuations are drawn from distributions with shape parameter θ, assumed the same for all regions $n \in N$. The value to household *h* of working in job $j|n$ is therefore $b(h,j|n)v_{j|n}$, and the proportion of the region's households choosing particular job $j'|n$ is $\pi_{j'|n} = v_{j'|n}^{\theta}/\sum_{j|n}^{L_i} v_{\,j|n}^{\,\theta}$. If all jobs in the region have the same base income, $v_{j|n} = v_n$ for all *j*, then this probability is the same for all jobs in the region, so probability and the expected utility derived from choice of job, u_n, take the form:

$$\pi_{j'|n} = v_n^{\,\theta}/\textstyle\sum_{j|n}^{L_n} v_n^{\,\theta} = 1/L_n, \qquad u_n = \Gamma\left(\frac{\theta-1}{\theta}\right)\left(\sum_{j|n}^{L_n} v_{\,j|n}^{\,\theta}\right)^{1/\theta} = v_n\left\{\Gamma\left(\frac{\theta-1}{\theta}\right) L_n^{1/\theta}\right\}. \qquad (2)$$

These expressions are analogous to those in (1), but applied at the level of jobs within a region, and with the additional feature that, since all jobs in the region give the same base income, summing gives $\sum_{j|n=1}^{L_i} v_n^{\,\theta} = L_n v_n^{\theta}$, so all jobs in the region are chosen with equal probability $1/L_n$.

The expected utility from this second stage choice, u_n (equation 2), depends on v_n and the term in curly brackets, $\left\{\Gamma(1-1/\theta)\, L_n^{1/\theta}\right\}$, which we term the expected match premium (EMP). As $\theta \to \infty$, all jobs are perceived to be perfect substitutes so the number on offer is immaterial and EMP = 1. Lower values of θ imply greater heterogeneity of individuals' valuations over jobs, so the quality of the job match matters. The key point is that this match is better, and expected utility higher, the more jobs are on offer in the place. The product of base income and EMP can be thought of as an indifference curve defined

[3] See appendix 1 for an outline of statistical theory underlying this and for references.

[4] Households take the number of jobs, L_n, as parametric. Its equilibrium value is derived in what follows.

over base income in a region and the number of jobs it offers; a region with a lot of jobs is attractive to workers even if the base income v_n is low.

At the first stage, households choose region, and this is done on the basis of the expected utilities from working in each region, u_n as given in equation (2), and idiosyncratic valuations of regions. The value to household h of choosing region n is then $b(h,n)u_n, n \in N$, the product of the expected utility of working in each region times households' idiosyncratic regional valuation $b(h,n)$, this distributed with shape parameter η. The probability of choosing region n is therefore $\Pi_n = u_n^\eta / \sum_{m=1}^N u_m^\eta$, which, using u_n from equation (2) gives

$$\Pi_n = u_n^\eta / \sum_{m=1}^N u_m^\eta = \left(v_n L_n^{1/\theta}\right)^\eta / \sum_{m=1}^N \left(v_m L_m^{1/\theta}\right)^\eta = v_n^\eta L_n^{\eta/\theta} / \sum_{m=1}^N v_m^\eta L_m^{\eta/\theta} \tag{3}$$

The expected utility associated with this choice is

$$U = \Gamma\left(\frac{\eta-1}{\eta}\right)\left(\sum_{m=1}^N u_m^\eta\right)^{1/\eta}, \; with\, u_m = \; v_m\left\{\Gamma\left(\frac{\theta-1}{\theta}\right)\, L_m^{1/\theta}\right\}. \tag{4}$$

Equation (3) is the central result, stating that the proportion of the population choosing each region depends on base income and the number of jobs, the relative importance of these two variables given by the distribution of idiosyncratic valuations over regions (shape parameter η) and jobs in each region (θ). The regional choice in (3) reduces to that in (1) as $\theta \to \infty$, while lower values of θ capture the idea that workers are attracted to places that offer a greater number of jobs.

Household valuations of jobs (and hence benefit of a good match) can accrue to households in different ways. One is direct utility – workers simply prefer some jobs to others, so $b(h,j|n)$ is a preference parameter. Another is that the value is monetary. This would be the case if productivity is specific to the match between household *h* and job *j*, is measured by multiplicative factor $b(h,j|n)$ over the base income, and is paid to the worker rather than held by the employer.

Regional equilibrium I:

Relationship (3) above gives the probability that households choose to live and work in region *n*, so describes labour supply in the region. To draw out its implications, descriptions for base income, v_n, and for labour demand are needed.

Household base income is nominal base wage[5] in the region, w_n, divided by the cost of living, which we assume to be increasing in the region's population, H_n, and express as $c_n = \bar{c}_n H_n^\rho,\ \rho \geq 0,\ \bar{c}_n$ a region specific parameter. This is underpinned by a land and housing market which we do not spell out in detail (see for example Duranton and Puga 2015), and the iso-elastic form allows for a simple parameterization of the relationship. It follows that $v_n = w_n/(\bar{c}_n H_n^\rho)$. Using this in the numerator of equation (3), the equilibrium number of households in region *n* is

$$H_n = H\Pi_n = H\frac{\left(w_n/\bar{c}_n H_n^\rho\right)^\eta L_n^{\eta/\theta}}{\sum_{m=1}^N \left(w_m/\bar{c}_m H_m^\rho\right)^\eta L_m^{\eta/\theta}}, \tag{5}$$

[5] I.e., wage net of EMP.

where total population is $H = \sum_{n=1}^{N} H_n$. It is convenient to rewrite this to give the base wage at which place *n* supports H_n households given that it has L_n jobs. Since this captures household labour supply we denote it w_n^H, and it takes the form

$$w_n^H = H_n^{\rho+1/\eta} L_n^{-1/\theta} K_n, \quad \text{where } K_n \equiv \bar{c}_n \left\{ \left(\sum_{m=1}^{N} \left(w_m / \bar{c}_m H_m^{\rho} \right)^{\eta} L_m^{\eta/\theta} \right) / H \right\}^{1/\eta}. \tag{6}$$

The terms grouped together in K_n are parameters and an economy wide sum, exogenous to a single small region.

For labour demand, we initially follow the usual approach in spatial equilibrium models of assuming that the nominal wage equals the average value product of labour, and the number of jobs adjusts to fully employ labour supply. A simple form assumes that labour is the only input to production and its value productivity is $a_n(L_n) = \bar{a}_n L_n^{\chi}$. The (inverse) labour demand curve is then

$$w_n^L = \bar{a}_n L_n^{\chi}, \tag{7}$$

where w_n^L is the base wage paid (superscript L indicating labour demand). Parameter χ measures returns to scale, and is positive if value productivity is increasing with employment. With this specification all jobs in region *n* pay the same base wage, depending on region-wide economies of scale.

$H_n = L_n$ is a necessary condition for equilibrium, in which case (6) and (7) can be written as

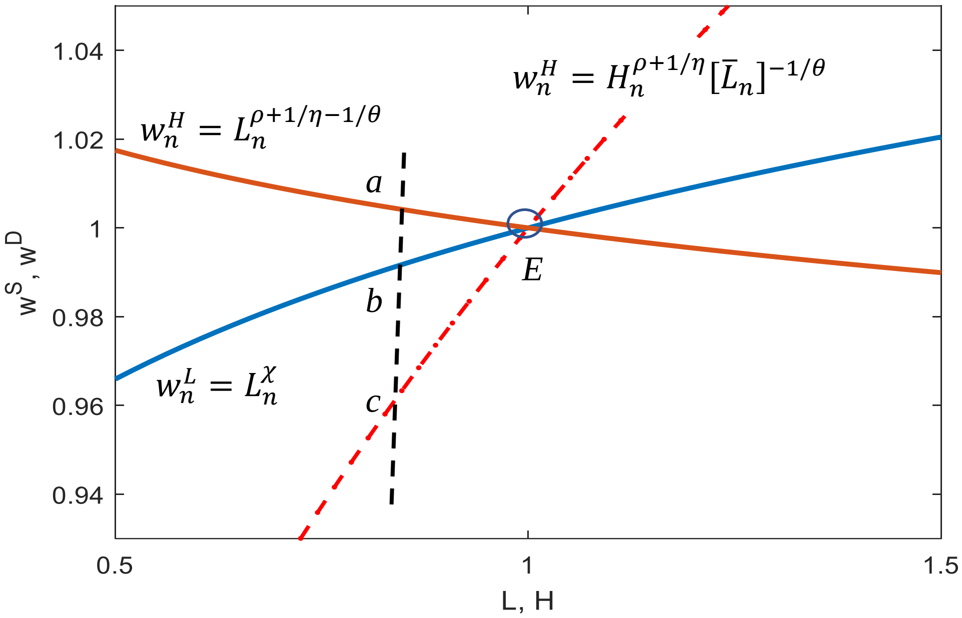
$$w_n^H = L_n^{\rho+1/\eta-1/\theta} K_n, \qquad w_n^L = \bar{a}_n L_n^{\chi}, \tag{8}$$

and equilibrium is the wage $w_n = w_n^H = w_n^L$ and employment level L_n satisfying this pair of equations.

Figure 1 has the number of households and jobs on the horizontal axis and the wage on the vertical, and the solid lines are the pair of equations in (8). The figure is constructed with $\chi > 0$, increasing returns (agglomeration economies) giving upwards sloping labour supply. The labour demand curve incorporates the fact that, if θ is finite, then labour supply is increasing in the number of jobs in the region, as well as with the real wage. The figure is constructed with θ small enough that the former effect dominates and labour supply slopes downwards $\rho + 1/\eta < 1/\theta$.

Point *E* is an equilibrium, but the fact that labour demand intersects labour supply from below suggests that this equilibrium unstable. For example, consider a perturbation such that $L_n = H_n$ at the value indicated by the black dashed line. If the (out of equilibrium) wage is anywhere in the interval (*a* ,*b*), then $w_n^H > w_n > w_n^L$. The first inequality suggest that households would leave the region, and the second suggests that firms and jobs would leave, pushing both variables further away from the equilibrium. In contrast, if the number of jobs is held constant, along the red dashed lines (equation (5) with $L_n$constant), the out of equilibrium wage might be expected to be in interval (*b*, *c*), with $w_n^H < w_n < w_n^L$. Absent the attraction force created by a large number of jobs, the equilibrium wold be stable.

Figure 2: Equilibrium in a single region*:*



Parameters: χ =0.05, $\rho = 0.1$, $\eta = 8$, $\theta = 4$, $K_n = 1$, $\bar{a}_n = 1$

In following sections we present a more formal treatment of stability, and the question of what a stable equilibrium might look like. However, the key intuition comes from the interaction of forces for dispersion and for agglomeration. The dispersion forces are $\rho + 1/\eta$, the first of these meaning that higher employment raises rents and the cost of living, and the second (small η) meaning the strong intrinsic valuations of places supports a dispersed population. The agglomeration forces are $\chi + 1/\theta$, the first being productivity effects of agglomeration, and the second a high value for job matching, attracting population to large labour markets. The argument above suggests that equilibrium is unstable if $\chi + 1/\theta > \rho + 1/\eta$.

We note three further points. First, this framework collapses to the standard one if $1/\theta = 0$. There is no variance in the valuation households assign to particular jobs, except that associated with their location. Second, if $1/\eta = 0$ then the object of choice is simply a job, and households care about location only in so far as it affects w_n and c_n. Third, agglomeration can occur without any of the technological agglomeration forces captured in χ, coming solely from workers' perceived benefit from being in a large labour market.

3. Firm location: labour demand

To this point we have, in line with the spatial equilibrium literature, assumed that firms are essentially passive, creating the number of jobs that are consistent with households' location choices. What if firms also have idiosyncratic valuations over the job matches they make, and over the regions in which they locate? We start by setting up their choices in a manner analogous to the preceding treatment of households. This assumes that matching decisions are taken by firms and that all households in a region receive the same real income, with the benefits of good matches accruing to firms. We then combine this with household choice, noting that job market matches cannot be the maximising choices of both sides of the market unless their idiosyncratic choices coincide perfectly.

We assume a two-stage (nested) process for firms, analogous to that for workers. At the second stage, conditional on locating in place *n*, each firms chooses workers. All workers in each place receive the same wage, w_n, and there is region-wide productivity level is a_n. In addition, there is match specific productivity effect $a(j,h|n)$, so output per unit wage cost is $a(j,h|n)a_n/w_n$. (As before, *j* indexes jobs, *h* households and *n* places). Valuations $a(j,h|n)$, are Frechet distributed with shape parameter β. Firms employ workers for whom the value of output produced per unit wage cost is highest (i.e. unit costs of production are minimised). If a_n and w_n are the same for all jobs in region *n*, then the probability of employing each worker is $1/H_n$ and the expected value of output per unit wage cost is, analogous to equation (2),

$$z_n = (a_n/w_n)\left\{\Gamma\left(\frac{\beta-1}{\beta}\right)H_n{}^{1/\beta}\right\} \tag{9}$$

The term in curly brackets is the expected match premium (EMP), but now accruing to firms, in contrast to the previous section where the EMP accrues to workers.

Firms may also have idiosyncratic preferences over regions, $a(j,n)$, and these are Frechet distributed with shape parameter α, so the proportion of firms choosing region *n* is $\Pi_n = z_n^{\eta}/\sum_{m=1}^{N} z_m^{\eta}$. Using equation (8) gives (analogous to equation (3)), choice of region as

$$\Pi_n = z_n^{\alpha}/\textstyle\sum_{m=1}^{N} z_m^{\alpha} = (a_n/w_n)^{\alpha}H_n^{\alpha/\beta}/\sum_{m=1}^{N}(a_n/w_n)^{\alpha}H_n^{\alpha/\beta}\ . \tag{10}$$

We suppose that region specific productivity depends on employment, as before (equation 7) so $a_n(L_n) = \bar{a}_n L_n^{\chi}$. Defining $D_n \equiv \bar{a}_n\left\{\sum_{m=1}^{N}(a_n/w_n)^{\alpha}H_n^{\alpha/\beta}/L\right\}^{1/\alpha}$ the total number of jobs in place *n*, $L\Pi_n$, is $L_n = L\Pi_n = w_n^{-\alpha}H_n^{\alpha/\beta}L_n^{\chi\alpha}D_n^{\alpha}$. Inverting this, the inverse labour demand curve is

$$w_n^L = H_n^{1/\beta}L_n^{\chi-1/\alpha}D_n. \tag{11}$$

w_n^L is the base wage (net of EMP) that is offered by employers in region *n* consistent with there being H_n households and L_n jobs. This labour demand curve is an extension of that in equation (7) and, as expected, a finite value of β is an agglomeration force, as large labour markets give better matches, while a finite value of α is a dispersion force, as firms have idiosyncratic region specific productivity differences.

Combining the decisions of both households and firms (equations (8) and (11)) gives the summary contained in Table 1. The Frechet approach gives four elasticities (shape parameters) and economic fundamentals two further elasticities, ρ, the elasticity of the regional cost of living with respect to population, and χ, agglomeration economies. While it is helpful to present these forces together, they may not all operate in any particular context. In particular, job matching rests on maximising choices being made by households (labour supply) and firms (labour demand). These choices may be perfectly aligned – if, for example, they are driven by productivity with shared benefits – so both sides make the same choice. But if they are not aligned then only one-side can secure its optimal choice, and this needs to be assigned, in some way (not addressed here), to either households or firms.

Table 1: Summary of household and firm choices

Households: labour supply: Equation 5: $w_n^H = H_n^{\rho+1/\eta} L_n^{-1/\theta} K_n$			Firms (jobs); labour demand: Equation 11: $w_n^L = H_n^{1/\beta} L_n^{\chi-1/\alpha} D_n$.		
	Idiosyncratic value	Shape parameter		Idiosyncratic value	Shape parameter
Worker h's valuation of match with job j in region n.	$b(h,j\|n)$	θ	Firm j's valuation of match with worker h in region n.	$a(j,h\|n)$	β
h's valuation of region n	$b(h,n)$	η	j's valuation of region n.	$a(j,n)$	α
Elasticity of cost of living wrt H		ρ	Elasticity of productivity wrt L		χ

Noting these qualifications, we undertake stability analysis carrying all four shape parameters through the remaindere of this section; following conditions may be interpreted with either $1/\beta = 0$ (households set the match) or $1/\theta = 0$ (firms set the match). Focusing on a particular small region, there are three state variables, H_n, L_n, and wage w_n. We look at dynamics in the neighbourhood of a symmetric equilibrium, in which parameters take values $\bar{a}_i = \bar{c}_n = 1$, $H = L = N$, and hence endogenous variables take equilibrium values $H_n = L_n = 1$, $w_n = 1$, and $K_n = D_n = 1$. Out of equilibrium we suppose that H_n and L_n adjust in line with the deviation between the actual wage and that respectively on the inverse labour supply and labour demand curves, so that H_n is increasing with w_n/w_n^S and L_n is increasing in w_n^D/w_n. With adjustment speeds δ_H, δ_L, and using equations (8) and (11) this gives

$$\hat{H}_n = \delta_H(w_n/w_n^H - 1) = \delta_H\left(w_n H_n^{-(\rho+1/\eta)} L_n^{1/\theta} - 1\right),$$

$$\hat{L}_n = \delta_L(w_n^L/w_n - 1) = \delta_L\left(w_n^{-1} H_n^{1/\beta} L_n^{\chi-1/\alpha} - 1\right), \tag{12}$$

where $(\widehat{\ \ })$ denotes proportionate change. Base wages in each place adjust according to excess demand for labour, so $\hat{w}_n = \delta_w(L_n/H_n - 1)$, where δ_w is the adjustment speed. Denoting deviations from symmetric equilibrium values by Δ, this gives the following system of differential equations.

$$\begin{bmatrix} \hat{H}_n \\ \hat{L}_n \\ \hat{w}_n \end{bmatrix} = \begin{bmatrix} -\delta_H(\rho + 1/\eta) & \delta_H/\theta & \delta_H \\ \delta_L 1/\beta & \delta_L(\chi - 1/\alpha) & -\delta_L \\ -\delta_w & \delta_w & 0 \end{bmatrix} \begin{bmatrix} \Delta H_n \\ \Delta L_n \\ \Delta w_n \end{bmatrix}. \tag{13}$$

The determinant of the coefficient matrix is $[\chi + 1/\beta + 1/\theta - \{\rho + 1/\alpha + 1/\eta\}]\delta_H\delta_L\delta_W$. If this is positive then at least one of the eigenvalues is positive.[6] Since there are no jump variables available to put the system on a stable manifold, the system is unstable if $\chi + 1/\beta + 1/\theta > \rho + 1/\alpha + 1/\eta$, exactly as suggested by the more restrictive analysis of Figure 1. As before, the left hand side of this inequality contains destabilising forces of agglomeration in production and now also the possible

[6] The determinant is the product of the three eigenvalues, so all three cannot be negative.

attraction of firms and households to large labour markets. The right hand side are the stabilising forces of land and house prices and the intrinsic preferences of households and firms for particular regions.

These results generalise the standard approach, capturing the intuitive notions that workers' location choices may be influenced by the number of jobs in a place, and firms' choices influenced by the number of workers. Unsurprisingly, these are forces for agglomeration, so they can destabilise an equilibrium in which employment is relatively evenly distributed across places. The forces can operate on either side of the market, with workers being attracted to places with jobs, or firms attracted to places with many workers.

Using this approach in different economic contexts, it is important to note that different agglomeration and dispersion forces operate at different spatial, sectoral, or occupational levels. For example, the cost of living (parameter ρ) is principally place specific; technical agglomeration economies (χ) are likely to be place and sector specific; matching effects (parameters θ, β) are likely to be place and occupation specific, although possibly not sector specific. We draw out some of the implications of these differences in the following section.

4. Regional equilibrium: Two occupations, two sectors, and two regions.

The argument that workers do not move to places with few jobs is expressed most often in terms of particular skill levels. Workers with specific non-transferable skills do not go to places with few jobs that require these skills, and should people in such a region acquire skills, then they are likely to choose to move to places with more jobs that match their skills. Furthermore, it seems plausible that the number of jobs on offer is more important to workers in some occupations – those with less transferable skills -- than in others. To capture these differences we extend the model of section 2 to two skill groups, and place this in a full general equilibrium setting with two productive sectors and two regions. We concentrated on job and location choice of households, so $\alpha, \beta \to \infty$.

The labour types are labelled by superscripts $s = A, B$, and valuations of jobs by each type are Frechet distributed with shape parameters θ^A, θ^B. The two skill analogue of equation (3), giving the choice of households with skill type-*s* across locations *n*, is

$$\Pi_n^s = \left((L_n^s)^{1/\theta^s} v_n^s\right)^{\eta} / \textstyle\sum_{m=1}^{N} \left((L_m^s)^{1/\theta^s} v_m^s\right)^{\eta}, \quad n \in N;\ s = A, B. \tag{14}$$

Importantly, workers of type-*s* care about the number of jobs of type-*s*, L_n^s, (not aggregate employment), and the shape parameters θ^A, θ^B for each type may differ. Both types are assumed to have preferences over regions distributed with the same shape parameter, η.

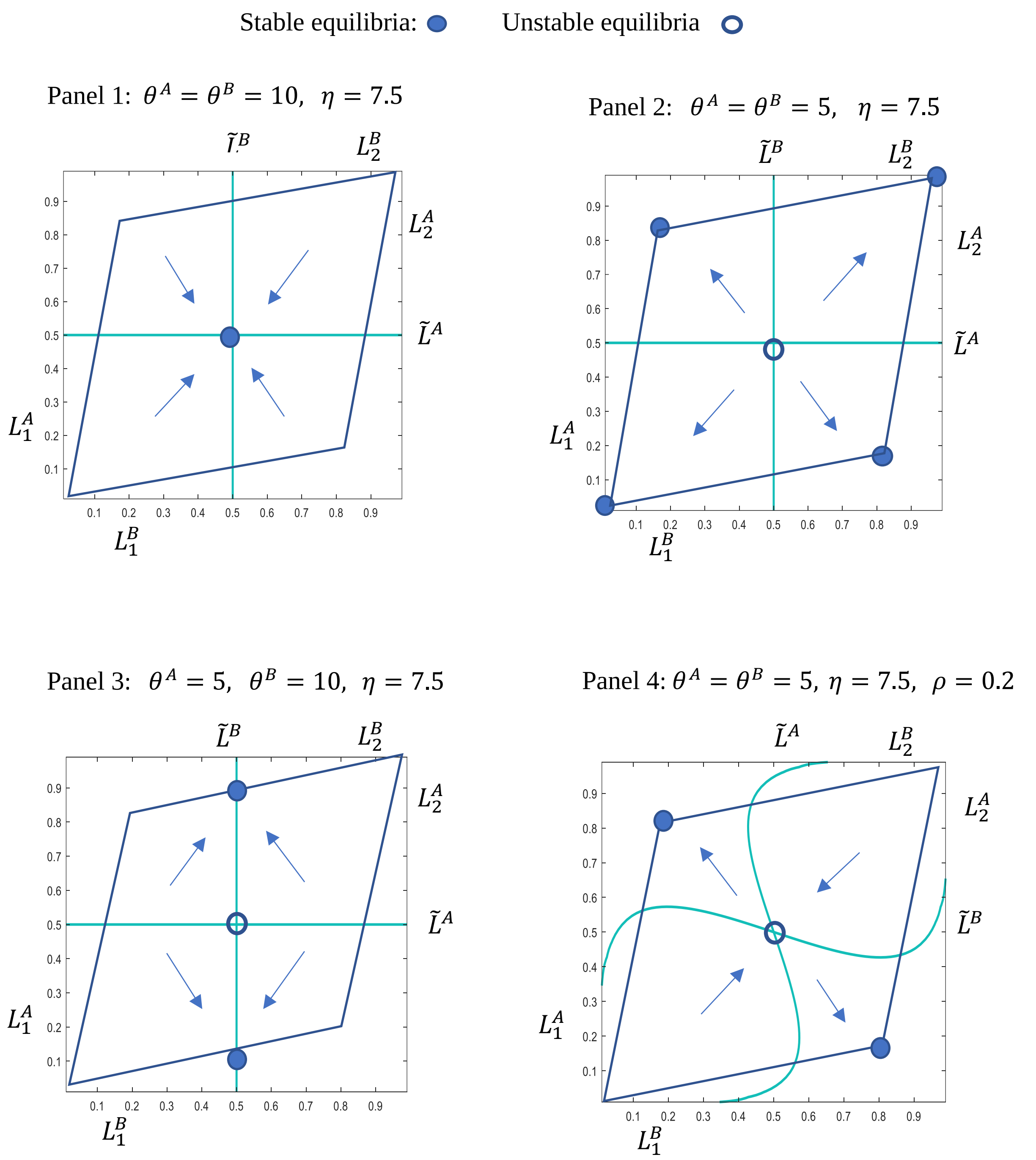
For the general equilibrium setting we suppose that the country under study is divided into two regions ($N = 2$) and endowed with fixed quantities of each of the two skill types, $H^s = \sum_{n=1,2} H_n^s,\ s = A, B$, so the division of households of type *s* between regions is given by $H_n^s = H^s \Pi_n^s,\ n = 1,2\ \ s = A, B$. The cost of living in each region is increasing in the resident population, so $c_n = \bar{c}(H_n^B + H_n^A)^{\rho}$. The economy has two production sectors, each using these skill types in different proportions. It is open to trade with the rest of the world at fixed prices, this having two implications. First, production and consumption decisions are separated, enabling analysis to focus on the production side. And second, if productivity levels are equal and constant in all regions, base wage rates for each skill type depend just on technology and world prices. These wages are the same in all regions where both sectors are active.[7] The number of jobs in each place and sector adjust such that the resident population is fully employed, $H_n^s = L_n^s,\ n = 1,2\ \ s = A, B$.

Figure 2 gives trade theory's standard diagram for analysing this. The bottom left corner of each panel of this figure is the origin from which region 1's stock of type-A skilled workers is measured on the horizontal axis, its stock of type-B on the vertical; the top right corner is the origin for region 2. The length and height of the box are respectively L^A, L^B, and a point in the box represents a division of workers of each skill type between the two regions. The slopes of the rays from each origin are the skill-intensities of the two sectors, and the parallelogram is the diversification set, i.e. the set of values $\{L_1^A, L_1^B, L_2^A, L_2^B\}$ which can be fully employed by non-negative scales of operation of the two sectors. Absent any job or region preferences ($\theta = \eta = \infty$) and with constant returns to scale and no cost of living effects ($\chi = \rho = 0$) any point inside the parallelogram (i.e. any division of skill types between regions) is an equilibrium. Industrial structure – the scale of operation of each sector in each region – adjusts to secure full employment at unchanged factor prices and factor intensities. The figure is constructed with symmetric technologies and skill endowments,[8] and on this blank canvas we now add job and location preferences, initially holding $\chi = \rho = 0$. If workers choose locations according to (14), what distributions of workers and jobs of each type between the two regions (i.e. points on the figure) are stable, and unstable equilibria?

The first panel of figure 2 describes the case where $\theta^A = \theta^B > \eta$, so preferences for regions are strong, relative to those for jobs. The horizontal line $\tilde{L}^A$ gives the division of the type-A labour force such that $\Pi_n^A = L_n^A/L^A,\ n = 1,2$, i.e. the division of the type-A labour force consistent with individual choices as given by equation (14). In terms of stability analysis, this can be thought of as the line along which L^A is stationary. At a fully symmetric equilibrium this is the case at $L_1^A = L_2^A = 1/2$. Since $\theta^A > \eta$ this is 'stable'; above the horizontal line $\tilde{L}^A$ household choices are such that $\Pi_1^A/\Pi_2^A < L_1^A/L_2^A$, so choice reduces the proportion of the type-A labour force in region 1 and increases it in region 2. The vertical line $\tilde{L}^B$ is the analogous expression for the type-B labour force, the two giving a stable equilibrium at their intersection. This is the unique equilibrium, supported by dispersion effects (low η) relative to agglomeration effects (high θ^A, θ^B).

[7] This is the factor price equalization theorem of international trade theory. See Dixit and Norman (1980) for generalization of the 2x2 case to higher dimensions.
[8] $L^A = L^B = 1$ and the relative *A* intensity of one sector is equal to the relative *B* intensity of the other. World prices of the two goods are equal, these assumptions jointly implying v_m^s is the same for all *s*, *m*.

Figure 2: Location and specialisation with two skill-types:



The second panel reverses this inequality, and has $\theta^A = \theta^B < \eta$. The symmetric equilibrium and lines $\tilde{L}^A, \tilde{L}^B$ are as before but, in line with the analysis of stability, directions of movement are reversed. The interior (symmetric) equilibrium is unstable, with directions of movement illustrated by the arrows. Job preferences create four stable equilibria. Two are with all activity in either region 1 or region 2. And two are with complete sectoral specialisation of each region, at the corners of the diversification set. Thus, the upper left corner of the parallelogram has all of the A-intensive sector taking place in region 1, and its skill mix, L_1^A/L_1^B equal to the skill-intensity of this sector. Region 2 has all of the B-intensive sector. At this equilibrium type-A workers are making better matches in region 1 than they are in region 2, so to support the equilibrium proportions, $\Pi_1^A/\Pi_2^A < L_1^A/L_2^A$, base wages w_1^A, w_2^A will differ between regions, as will those of type-B workers.[9]

Panel 3 takes the case where job preferences of each skill type lie on different sides of region preferences, $\theta^A < \eta < \theta^B$, with job preference stronger for type-A labour. Directions of movement are now away from the symmetric equilibrium for type-A labour, and towards it for type-B. The interior equilibrium is unstable and the two stable equilibria are as indicated.[10] At the upper one all of the A-intensive sector is concentrated in region 1, and with it most of the type-A skilled workers (with $\theta^A < \eta$). However, type-B labour has relatively strong location preferences, $\eta < \theta^B$, so is split between regions. Region 2 is fully specialised in production of the B-intensive good, but region 1 is not fully specialised, also having some employment in the B-intensive sector.

Finally, panel 4 takes the same values $\theta^A = \theta^B < \eta$ as panel 2, but adds the dispersion force of $\rho > 0$. While nominal wages remain fixed from the production side of the economy, the cost of living and hence v_m^s now depend on the total labour force in each region. This has the effect of rotating the stationaries, with both lying on the main diagonal when $1/\theta = \rho + 1/\eta$ (with $\chi = 0$ this is the boundary between stability and instability as established in the preceding section). The direction of intersection of the two stationaries is reversed, compared to other panels, for large enough ρ. The outcome is intuitive. Concentration of all activity in a single region ceases to be an equilibrium, as the cost of living differential between regions becomes too high. In the stable equilibria the two regions are the same size (so have the same cost of living), but their production is fully specialised, and workers of each skill type are located in a place that is fully specialised in the sector intensive in their skill.

All panels of Figure 2 are constructed with productivity held constant ($\chi = 0$). If agglomeration economies operate within-sector to create Hicks neutral technical change (so sector productivity is an increasing function of sector output), then they are equivalent to a region specific output price change. The insights of the Stolper-Samuelson theorem then mean that, within the diversification set, an increase in sector output raises the wage of the skill-type used intensively in that sector and reduce the wage of the other skill-type. This magnifies forces for sectoral concentration, as would be expected.

[9] The four wages w_n^s are determined by four equations, two aligning choice probabilities to employment levels, and two aligning unit production costs to output price. Since each region is specialised in one good, the unit cost of the other good weakly exceeds price. Appendix 2 gives the full set of equations characterising equilibrium.

[10] The $\tilde{L}^A = 0$ stationary is a saddlepath, but there is no jump variable or foresight mechanism suggesting any reason why location choices should follow this path.

While the reasoning behind panels 1 and 2 is in line with the homogenous labour case, that behind panels 3 and 4 gives two further insights. First (panel 3), the symmetric equilibrium is destabilised by strong job preferences in just one type of labour; it does not require it for all skill types. This insight carries through to multi-sector and multi-skill type models, as the mechanism destabilising the symmetric equilibrium is the concentration of one or more sector in one place. Second, (panel 4), while the dispersion force of the land price and cost of living works against complete concentration of all workers in one region it has a much weaker effect on sectoral concentration, since each region gets an entire sector, whilst having similar levels of population and thence land prices. The agglomeration forces are based on labour market matching, and hence depend on the skill composition of different sectors and thence regions. This is quite different from the overall or sectoral output or employment levels assumed to drive Marshall-Romer agglomeration.

5. Objects of choice; jobs, places of work and places of residence.

Quantitative spatial models have been widely applied in the intra-urban context, in which commuting is possible so households face choice of both place of residence and place of work. The approach of this paper is readily applied to this context, and in this section we sketch possible ways of modelling these choices.

As before there are N places, now interpreted as areas within the city. All of these can serve as place of work (subscripted n, m) and as place of residence (subscripted r, s). The base income derived from working in n and living in r is v_{nr}. This is based on three elements, a base wage in place of work, v_n, a cost of living in place of residence, c_r, and a commuting cost for travel between them, $\tau_{nr} \geq 1$. These combine multiplicatively so that $v_{nr} = w_n / c_r \tau_{nr}$.

The standard, single stage, approach in the literature is to suppose that the object of choice is a residence/ workplace pair. Choice is based on the value of v_{nr}, with Frechet distributed idiosyncratic preferences $b(h, r \times n),\ r, n \in N$ over the N^2 pairs of place (work and residence) with shape parameter η, so the proportion of the population working in particular place n and living in r is

$$\Pi_{nr} = v_{nr}^{\eta} / \textstyle\sum_{s=1}^{N} \sum_{m=1}^{N} v_{sm}^{\eta}, \quad r, n \in N. \tag{15}$$

This is analogous to (1), although there are now N^2 options and the shape parameter describes the distribution of quite different valuations.[11]

Adding jobs a choice as an object of choice is also analogous to earlier sections, and extends (15) to

$$\Pi_{nr} = v_{nr}^{\eta} L_n^{\eta/\theta} / \textstyle\sum_{s=1}^{N} \sum_{m=1}^{N} v_{sm}^{\eta} L_m^{\eta/\theta}, \quad r, n \in N. \tag{16}$$

A finite value of shape parameter θ is, as before, a force for clustering of jobs.

The recursive structure of these choices make it straightforward to generalise this. For example, suppose that households choose job, place of work, and place of residence in nested sequence, i.e. choice of job (final stage, shape parameter θ), choice of work place (shape parameter η) and choice of

[11] In this section we continue to use shape parameters η, and for preferences over jobs, θ, although they will generally take different values from preceding sections.

place of residence (shape parameter ϵ). In this case the proportion of those residing in *r* who work in *n* is $\Pi_{n|r}$, analogous to (3) (see appendix 3 for derivation).

$$\Pi_{n|r} = v_{nr}^{\eta} L_n^{\eta/\theta} / \sum_{m=1}^{N} v_{mr}^{\eta} L_m^{\eta/\theta} .. \tag{17}$$

Associated with this is the utility of residing in *r*, this generating the probability of residing in *r*, (appendix 2),

$$\Pi_r = \left(\sum_m v_{mr}^{\eta} L_m^{\eta/\theta}\right)^{\frac{\epsilon}{\eta}} \Big/ \sum_r \left(\sum_m v_{mr}^{\eta} L_m^{\eta/\theta}\right)^{\frac{\epsilon}{\eta}}. \tag{18}$$

It follows that the proportion of the population working in *n* and residing in *r*, Π_{nr}, is given by

$$\Pi_{nr} = \Pi_{n|r}\Pi_r = \frac{\left(\sum_m v_{mr}^{\eta} L_m^{\eta/\theta}\right)^{\frac{\epsilon}{\eta}}}{\sum_r \left(\sum_m v_{mr}^{\eta} L_m^{\eta/\theta}\right)^{\frac{\epsilon}{\eta}}} \cdot \frac{v_{nr}^{\eta} L_n^{\eta/\theta}}{\sum_m v_{mr}^{\eta} L_m^{\eta/\theta}}. \tag{19}$$

This collapses to the combined choice, (16), if $\epsilon = \eta$, and to the standard approach (15) if $\theta \to \infty$. If people have more dispersed valuations over place of residence than place of work ($\epsilon < \eta$, then this nested approach gives more concentrated employment.

Finally, the seminal work by McFadden (1978) on housing choice adopted developed a nested approach to model a specific dwelling as the second stage object of choice, nested in choice of urban neighbourhood. Doing so would make neighbourhoods with a large housing stock relatively attractive, a clustering force analogous to the nesting of jobs analysed in this paper.

5. Concluding comments

The idea that people want to go to where the jobs are is intuitive, yet is absent from the standard modelling approach in which location choices are guided by prices, without reference to quantities (the number of jobs in a place). The purpose of this paper is to fill this gap by making jobs, as well as places, the objects of household choice. Making this change involves minor change to the modelling approach widely used in the literature, and provides a simple description of labour market matching. It yields substantially more powerful agglomeration forces, as workers' location choices become positively influenced by the number of jobs on offer in a place, as well as by the wages these jobs offer. Spatial disparities can arise, in the form of specialisation and concentration in a regional model or steep rent and density gradients in an urban model, even if there is no spatial variation in productivity or amenities.

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Appendix 1: Discrete choice:

This appendix outlines and provides references to the choice theory employed in the paper.

1: Choice probabilities: Results follow directly from reference to the multinomial logit, based on alternatives n yielding utility $u_n = y_n + g(n)$, where y_n is income and $g(n)$ are additive preference shocks drawn from a Gumbel distribution with standard deviation μ. The probability that n yields the highest utility is $\Pi_n = exp(y_n/\mu)/\sum_{m\in N} exp(y_m/\mu)$.

This is equivalent to a multiplicative shock drawn from a Frechet distribution with utility $u_n = v_n b(n)$ as can be seen by making substitution $v_n = \exp(y_n)$, this giving proportion of the population choosing n given by $\Pi_n = v_n^{\eta} /\sum_{m=1}^{N} v_m^{\eta}$ where the shape parameter is $\eta = 1/\mu$.

2: Expected utility: The distribution of maximum values of u is itself Frechet distributed with shape parameter η and scale parameter $(\sum_{m=1}^{N} v_m^{\eta})^{1/\eta}$. The pdf of this Frechet distribution gives the probability of receiving u, $f(u) = \eta u^{-(1+\eta)}\, \Phi e^{-\Phi u^{-\eta}}$, where $\Phi \equiv \sum_{m=1}^{N} v_i^{\eta}$. The expected value is therefore $E(u) = \int_0^{\infty} u\{\eta u^{-(1+\eta)}\, \Phi e^{-\Phi v^{-\eta}}\}du$. To integrate, change variable to $y = \Phi u^{-\eta}$, i.e. $u = (y/\Phi)^{-\frac{1}{\eta}}$ so that $du = -\left[\Phi^{\frac{1}{\eta}} y^{-\frac{1}{\eta}}/\eta\right] dy$ and hence $E(v) = \Phi^{\frac{1}{\eta}} \int_0^{\infty} y^{-\frac{1}{\eta}} e^{-y}\, dy \; = \Gamma\left(1 - \frac{1}{\eta}\right) \Phi^{\frac{1}{\eta}}$, by definition of the gamma function. (Notice that limits of integration change, cancelling out the minus sign).

3: Nesting: The nesting of several levels of decision making is standard in nested multinomial logit. This paper follows exactly analogous procedure with the Frechet distribution, using its choice probabilities and expected utility.

The reliance on extreme value distributions to model individuals’ choices is not quite as arbitrary as it seems. There are ‘extreme value theorems’ that parallel the ‘central limit theorem’. Loosely, if x is a random variable that is a maximum over a many draws of some other random variable y, then we expect x to have a Frechet or Gumbel distribution, or a similar third distribution, depending on the characteristics of the distribution on y. References include Anderson, de Palma and Thisse (1992) for nested multinomial logit, in particular chapter 2. For the Frechet distribution and expected utility, Ahlfeldt et al. (2015), in particular section 2.1 of the technical appendix. A useful source for gaining a quantitative sense of the distribution of the maxima of samples drawn from a Frechet distributions is https://www.acsu.buffalo.edu/~adamcunn/probability/frechet.html#:~:text=The%20maximum%20of%20n%20iid,distribution%20but%20not%20the%20shape.

Appendix 2: Two-skill, two-sector, two region model: (figure 2)

Full employment of each labour type: $L_1^A + L_2^A = L^A, \quad L_1^B + L_2^B = L^B$. (A2.1, A2.2)

Symmetric fixed coefficient technology with sector *X* A-intensive, $\omega > 1/2$.

Sector *X* inputs of type-A, type-B labour per unit output, $\omega,\ (1-\omega)$.

Sector *Y* inputs of type-A, type-B labour per unit output $(1-\omega),\ \omega$.

Figure 2 panel 3: Region 1 produces good *X* and region 2 produces good *Y*.

Price relative to unit cost:

In region 1: $p_X = \omega w_1^A + (1-\omega)w_1^B, \quad p_Y \leq (1-\omega)w_1^A + \omega w_1^B,$ (A2.3)

In region 2: $p_X \leq \omega w_2^A + (1-\omega)w_2^B, \quad p_Y = (1-\omega)w_2^A + \omega w_2^B$ (A2.4)

Factor demands (ratio): $L_1^A/L_1^B = \omega/(1-\omega), \qquad L_2^A/L_2^B = (1-\omega)/\omega$ (A2.5, A2.6)

Household location choice (in ratio form) (equation 14 with $v_n^s = w_n^s$)

$$\left(L_1^A/L_2^A\right)^{(1/\eta - 1/\theta^A)} = w_1^A/w_2^A, \qquad \left(L_1^B/L_2^B\right)^{(1/\eta - 1/\theta^B)} = w_1^B/w_2^B. \qquad \text{(A2.7, A2.8)}$$

This is a system of 8 equations in 8 unknowns, $w_1^A, w_1^B,\ w_2^A,\ w_2^B,\ L_1^A, L_1^B,\ L_2^A,\ L_2^B$.

If $\omega > 1/2$ and $1/\eta - 1/\theta^A < 0$, the assumed pattern of specialisation satisfies inequalities in (A2.3, A2.4) . It has the properties that $L_1^A/L_2^A = L_2^B/L_1^B > 1$, hence $w_1^A/w_2^A = w_2^B/w_1^B < 1$. Using this with $\omega > 1/2$ in A2.3, A2.4 establishes the inequalities.

Figure 2 panel 4: Region 1 produces both goods, and region 2 just produces good *Y*.

Price relative to unit cost:

In region 1: $p_X = \omega w_1^A + (1-\omega)w_1^B, \quad p_Y = (1-\omega)w_1^A + \omega w_1^B,$ (A2.9, A2.10)

In region 2: $p_X \leq \omega w_2^A + (1-\omega)w_2^B, \quad p_Y = (1-\omega)w_2^A + \omega w_2^B$ (A2.11)

Factor demands (ratio):

$$\frac{L_1^A}{L_1^B} = \frac{\omega X_1 + (1-\omega)Y_1}{(1-\omega)X_1 + \omega Y_1} = \frac{\omega + (1-\omega)Y_1/X_1}{(1-\omega) + \omega Y_1/X_1}, \qquad \frac{L_2^A}{L_2^B} = \frac{(1-\omega)}{\omega}$$

This is now 9 equations (additional A2.10), with the extra variable Y_1/X_1. It collapses to the previous case if $Y_1 = 0$. The two regions are no longer symmetric, but wage relativities $w_1^A/w_2^A < 1$, $w_2^B/w_1^B < 1$ hold and hence so does the inequality in A2.11.

Appendix 3: Choices in the urban model

Equations (3) and (4) from section 2 give the probability of choosing place n

$$\Pi_n = u_n^\eta / \textstyle\sum_{m=1}^N u_m^\eta = \left(v_n L_n^{1/\theta}\right)^\eta / \sum_{m=1}^N \left(v_m L_m^{1/\theta}\right)^\eta = v_n^\eta L_n^{\eta/\theta} / \sum_{m=1}^N v_m^\eta L_m^{\eta/\theta} \qquad (3)$$

and the expected utility associated with this choice is

$$U = \Gamma\left(\tfrac{\eta-1}{\eta}\right)\left(\textstyle\sum_{m=1}^N u_m^\eta\right)^{1/\eta}, \ \ with\ u_m = v_m\left\{\Gamma\left(\tfrac{\theta-1}{\theta}\right) L_m^{1/\theta}\right\}. \qquad (4)$$

Allowing place of residence to differ from place of work replaces v_n by $v_{nr} = w_n/c_r\tau_{nr}$. For a household resident in place *r*, the choices and utility levels given by (3) and (4) can be rewritten as

$$\Pi_{n|r} = v_{nr}^{\eta} L_n^{\eta/\theta} / \sum_{m=1}^{N} v_{mr}^{\eta} L_m^{\eta/\theta} \quad \text{and}$$

$$U_r = \Gamma\left(\frac{\eta-1}{\eta}\right)\left(\sum_{m=1}^{N} u_{mr}^{\eta}\right)^{1/\eta} \text{ where } u_{mr} = v_{mr}\left\{\Gamma\left(\frac{\theta-1}{\theta}\right) L_m^{1/\theta}\right\}.$$

Choice of place of residence *r* is now derived in the usual way, so

$$\Pi_r = U_r^{\epsilon}/\sum_r U_r^{\epsilon} = \left(\sum_m v_{mr}^{\eta} L_m^{\eta/\theta}\right)^{\frac{\epsilon}{\eta}} \Big/ \sum_r \left(\sum_m v_{mr}^{\eta} L_m^{\eta/\theta}\right)^{\frac{\epsilon}{\eta}}.$$

The proportion of the population working in *n* and living in *r* is then

$$\Pi_{nr} = \Pi_{n|r}\Pi_r = \frac{\left(\sum_m v_{mr}^{\eta} L_m^{\eta/\theta}\right)^{\frac{\epsilon}{\eta}}}{\sum_r \left(\sum_m v_{mr}^{\eta} L_m^{\eta/\theta}\right)^{\frac{\epsilon}{\eta}}} \cdot \frac{v_{nr}^{\eta} L_n^{\eta/\theta}}{\sum_m v_{mr}^{\eta} L_m^{\eta/\theta}}.$$