Technical change, jobs, and wages in the global economy

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Abstract

This paper presents a compact and intuitive framework that consolidates, simplifies, and extends results on the links between technology, trade, and labour market outcomes. It makes three main contributions. First, it presents closed-form solutions for the impacts of different types of technical change (TC) on jobs (the sectoral allocation of employment) as well as on wages, prices and output. Second, it shows that wage and employment effects are positively correlated only for certain types of TC and certain parameters, so wage and employment impacts need to be examined separately. Third, we incorporate a non-traded sector into our framework and show how employment in this sector alters results by offering a new margin of adjustment. The impact of TC on relative wages is dampened, although its sign is not changed. However, the signs of TC's impact on relative prices, outputs, and jobs can be reversed by inclusion of a non-traded sector.

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1. Introduction

Technological change (TC) and globalisation have been drivers of aggregate prosperity, but also of labour market disruption. In recent decades, technological advances have tended to favour more skilled versus less skilled workers (Acemoglu and Restrepo, 2018a,b) at the same time as downward trends in world prices for unskilled-intensive goods undermined unskilled workers' jobs and wages in advanced economies (Caliendo et al. 2019).

Research on these issues has been dominated by empirical work, with less theoretical research on the economic mechanisms underpinning the technology-trade-labour link. However, very recent developments in technology and globalisation suggest that theoretical, ex ante work may be necessary to understand how future labour outcomes may differ from the empirically observed outcomes of the past. AI-trained software, like ChatGPT, has begun to use data-based pattern recognition to automate knowledge-based pattern-recognition that is so critical to the productivity of high-skilled workers. For example, Noy and Zhang (2023) found that ChatGPT significantly improved the writing skills of less skilled writers, as compared to their more skilled counterparts. By automating jobs that used to require years of education and experience, such AI may increasingly disfavour skilled workers relative to unskilled workers, reversing the skill-biased technical change (SBTC) of the past. At the same time, some emerging-market exporters like China are raising their productivity in higher-end sectors like electric vehicles, commercial aircraft, and semiconductors.

Our paper consolidates, simplifies, and extends the study of the theoretical links between technology, trade, and labour, contributing to the existing body of theoretical work in three main ways. First, informed by insights originally presented by Ron Jones (Jones, 1965), we offer an intuitive and comprehensive framework for analysing the general equilibrium connections between openness, technology, jobs, and wages. The framework, based on the Heckscher-Ohlin model, uses the Jones-inspired correspondence between TC and relative factor-endowment changes to develop novel insights and to structure intuition for existing TC and labour results. We use it to present closed-form solutions for the effects of factor augmenting TC on equilibrium values of key variables. The analysis covers cases in which the TC is experienced in both larger and smaller proportions of the world economy, thereby affecting world prices to a different extent. Our comparative static analysis of the 2 -by-2 Heckscher-Ohlin model with varying degrees of goods price flexibility is more general than any in the literature of which we are aware.[1]

Second, the paper provides an integrated analysis of impacts of trade and technology on jobs as well as on wages, prices and output. The 'Jones Equivalent' framework allows us to derive simple, closed-form solutions for the impacts of trade and technology on the sectoral allocation of employment as well as wages. The focus of much of the existing literature on either jobs (allocation of workers across sectors) or wages, but not both, may have been due to the natural presumption that wage and employment effects would be so tightly aligned as to make results on jobs redundant to results on wages. For example, SBTC that raised skilled workers' relative wages could be expected to create jobs for them in skill-intensive sectors and destroy them in other sectors. We show that wage and employment effects are not necessarily positively related. A simple closed-form expression allows us to determine how the relationship depends on the underlying tastes, technology parameters, the nature of the TC, and the openness of the economy.

[1] The most comprehensive paper is Xu (2001) who presents sign conditions but not explicit solutions. General multi-sector results are in Dixit and Norman (1980) and Bound and Johnson (1992), but are not solved in explicit form. For a recent survey of this and related material see Hotte et al. (2023).

Third, while most workers in advanced economies work in non-traded sectors, and many of the low-skill workers displaced by technology and globalisation in recent decades have moved to non-traded sectors, the theoretical literature has not considered how the presence of a non-traded sector alters the linkages between trade, technology, and labour market outcomes. To fill this lacuna, we extend our framework to include a non-traded sector and show how its inclusion affects standard results. Although the sign of TC's wage effect is not altered, we show that the strength of the connection is dampened for all types of TC and relative factor intensities. By contrast, the impact of TC on relative prices, outputs, and jobs can change sign for some forms of TC and some parameter values.

Since the analytic framework we employ is based on the Jones Equivalent between TC and relative endowment changes, it can also be used to study the impact of exogenous migration shocks on relative price, output, wage, and employment. We do not pursue this explicitly but do include relative endowment changes in the closed-form solutions, so the application to migration shocks is straightforward.

The paper has 6 sections following the introduction. Sections 2, 3 and 4 present the basic model and how it can be used to study the impacts of TC. Section 5 considers restricted, but widely discussed, forms of TC, namely, pure factor-biased TC (such as SBTC), and pure sector-biased TC. Section 6 presents our extension of the model that allows for a non-trade sector and the final section contains a short summary of the results and our concluding remarks.

2. The baseline model

We work with the familiar Heckscher-Ohlin (HO) model and so move quickly after introducing the notation. Referring to the two factors as more skilled labour (factor A) and less skilled labour (factor B), and the sectors as 1 and 2, we denote sectors and factors with subscripts $s = 1,2$ and $f = A, B$, respectively. By convention and without loss of generality, we assume that sector 1 is relatively intensive in factor A (skilled labour), while sector 2 is relatively intensive in factor B (unskilled labour). Factor endowments, L_A and L_B, are exogenous, competition is perfect in all markets, and returns to scale are constant in both sectors. We assume identical and homothetic preferences for representative consumers. We also assume free trade throughout our model, along with standard regularity conditions ensuring all nations produce both goods.[2]

Technology is defined by unit cost functions, $c_s(w_A/\alpha_{As}, w_B/\alpha_{Bs})$, and production functions, $X_s = F_s(\alpha_{As}L_{As}, \alpha_{Bs}L_{Bs})$, where w_f is the wage of factor f. We work with factor-augmenting TC that enters via the parameters α_{fs}. X_s is sector-s output, and L_{fs} is employment of factor f in sector s. By way of terminology, $\alpha_{fs}L_{fs}$ and w_f/α_{fs} are, respectively, the effective units of L_f employed and the effective wage of factor f in sector s. The pricing, production, and market-clearing conditions are:

$$p_s = c_s(w_A/\alpha_{As}, w_B/\alpha_{Bs}), \qquad s = 1,2, \tag{1}$$

$$X_s = F_s(\alpha_{As}L_{As}, \alpha_{Bs}L_{Bs}), \qquad s = 1,2, \tag{2}$$

$$L_f = L_{f1} + L_{f2}, \qquad f = A, B, \tag{3}$$

$$p_1/p_2 = (X_1/X_2)^{-1/\varepsilon}, \qquad \varepsilon = \big((1-\delta)\,\eta^* + \varepsilon^o\big)/\delta. \tag{4}$$

These equations describe a country, or integrated set of countries, experiencing common TC and trading freely within the set (which we will refer to as 'home') and outside. ε^o is consumers' price elasticity

[2] Specifically, the factor endowment ratios must lie in the space spanned by the two sectors' equilibrium factor intensity ratios.

of relative demand, and ε is the inverse of the responsiveness of the world relative price to relative output produced by home. This depends on the home's share of global output, δ, and the rest-of-world general equilibrium supply responsiveness, η^*. The formula for the response of relative price to relative output, ε, in (4) follows from the global relative demand and supply equality, $(X_1 + X_1^*)/(X_2 + X_2^*) = D(p_1/p_2)^{-\varepsilon^o}$, where X_s^* is rest-of-world output in sector-s. Using a reduced form relationship, $X_1^*/X_2^* = S(p_1/p_2)^{\eta^*}$, to capture the rest-of-world supply response, log differentiating yields $\varepsilon \approx (1/\delta)[(1-\delta)\,\eta^* + \varepsilon^o]$, this being exact, as given in (4), when home's share of global output is the same for both goods (see Appendix 1 for details). This approach embeds i) the small open economy case where $\delta = 0$, so $\varepsilon = \infty$ and prices are unresponsive to home output, ii) the closed economy case where $\delta = 1$, so $\varepsilon = \varepsilon^o$, and iii) the large open economy case where $0 < \delta < 1$, so $\varepsilon^o < \varepsilon < \infty$. Since δ can be interpreted as the share of the world economy experiencing the TC, $\delta = 1$ is the case in which the entire integrated world economy experiences the change. This approach to ε allows us to consider several special cases that are often considered separately in the literature.

Log differentiating (1)-(4) yields the standard 'equations of change':

$$\hat{p}_s = \omega_s(\hat{w}_A - \hat{\alpha}_{As}) + (1-\omega_s)(\hat{w}_B - \hat{\alpha}_{Bs}), \qquad s = 1,2, \tag{5}$$

$$\hat{X}_s = \omega_s(\hat{L}_{As} + \hat{\alpha}_{As}) + (1-\omega_s)(\hat{L}_{Bs} + \hat{\alpha}_{Bs}), \qquad s = 1,2. \tag{6}$$

$$\hat{L}_f = \upsilon_f \hat{L}_{f1} + (1-\upsilon_f)\hat{L}_{f2}, \qquad f = A, B. \tag{7}$$

$$\hat{X}_1 - \hat{X}_2 = -\varepsilon(\hat{p}_1 - \hat{p}_2). \tag{8}$$

where "^" indicates proportional change, ω_s is factor-A's cost share in sector s, and υ_f is sector-1's employment share of factor f:[3]

$$0 < \omega_s \equiv w_A L_{As}/p_s X_s < 1, \qquad 0 < \upsilon_f \equiv L_{f1}/L_f < 1, \qquad f = A, B, \;\; s = 1,2. \tag{9}$$

The elasticities of substitution in effective factor usage are:

$$\sigma_s \equiv -\left[\frac{(\hat{L}_{As} + \hat{\alpha}_{As}) - (\hat{L}_{Bs} + \hat{\alpha}_{Bs})}{(\hat{w}_A - \hat{\alpha}_{As}) - (\hat{w}_B - \hat{\alpha}_{Bs})}\right], \qquad s = 1,2 \tag{10}$$

where $\hat{L}_{fs} + \hat{\alpha}_{fs}$ is the change in the effective units of L_{fs}, so $(\hat{L}_{As} + \hat{\alpha}_{As}) - (\hat{L}_{Bs} + \hat{\alpha}_{Bs})$ is the change in the effective factor proportions in sector s, and $(\hat{w}_A - \hat{\alpha}_{As}) - (\hat{w}_B - \hat{\alpha}_{Bs})$ is the change in the sector-specific relative effective wage. To simplify, we take $\sigma_1 = \sigma_2 = \sigma$.

Endowment and technology shocks. Using the operator Δ_{AB} to indicate differences in proportional changes across factors, and taking (by convention and without loss of generality) the change in A minus the change in B, the classic relative factor abundance shock is:

$$\Delta_{AB}\hat{L} \equiv \hat{L}_A - \hat{L}_B. \tag{11}$$

For instance, if $\Delta_{AB}\hat{L} > 0$, the supply of skilled labour rises relative to the supply of unskilled labour.

Many forms of TC can be studied by allowing the $\hat{\alpha}_{fs}$ to vary by sector and factor, including factor-biased (e.g. skill-biased) technical change, sector-biased technical change, Hicks neutral TC, and TC

[3] The υ_f and ω_s are linked as both reflect factor-intensity difference, specifically, $\upsilon_A = s\omega_1/\bar{\omega}$ and $\upsilon_B = s(1-\omega_1)/(1-\bar{\omega})$ where s is the share of sector 1 in GDP, and $\bar{\omega} \equiv s\omega_1 + (1-s)\omega_2$ is the average cost share of factor-A in the economy.

involving particular factors in specific sectors. Nevertheless, the HO structure naturally points to a focus on two collections of TC shocks. Expression (5) shows that the price change consists of cost-share weighted changes in wages minus TC-induced cost savings by sector. We denoted the latter as:

$$\hat{\chi}_s \equiv \omega_s \hat{\alpha}_{As} + (1 - \omega_s)\hat{\alpha}_{Bs}, \qquad s = 1,2, \tag{12}$$

where chi is a mnemonic for 'cost savings'. The second natural collection is the TC-induced labour saving by factor suggested by (7):

$$\hat{\lambda}_f \equiv \upsilon_f \hat{\alpha}_{f1} + \left(1 - \upsilon_f\right)\hat{\alpha}_{f2}, \qquad f = A, B, \tag{13}$$

where lambda is a mnemonic for 'labour savings'.

Measures of job creation/destruction. A key focus of our paper is on job creation/destruction. Since all factors are employed in equilibrium, job creation/destruction is defined at the sectoral level. For example, if the question is whether skill-biased TC creates jobs for skilled workers, the answer must be sector specific since all skilled workers will be employed. This seems to line up with the thinking of empiricists seeking to determine, for example, whether robots in the manufacturing sector create or destroy jobs in the manufacturing sector (Aghion et al. 2022). By convention and without loss of generality, we track sector-level job creation/destruction by focusing on the shift of factor-f jobs to sector 1 from sector 2. We denote this by:

$$\Delta_{12}\hat{L}_f \equiv \hat{L}_{f1} - \hat{L}_{f2}, \tag{14}$$

where Δ_{12} indicates the difference in proportional changes between sector 1 and 2. Note that full employment and given endowments imply that $\text{sign}\left(\Delta_{12}\hat{L}_f\right) = \text{sign}\left(\hat{L}_{f1}\right) = -\text{sign}\left(\hat{L}_{f2}\right)$, so signing $\Delta_{12}\hat{L}_f$ pins down the sign of employment changes in each sector.

Difference in difference (DiD) equations. The core HO insights concern relative changes – a point that comes out starkly when the equations-of-change are differenced across factors and sectors to yield the difference-in-difference (DiD) equations. Introducing the notation:

$$\Delta_{12}\hat{p} \equiv \hat{p}_1 - \hat{p}_2, \qquad \Delta_{12}\hat{X} \equiv \hat{X}_1 - \hat{X}_2, \qquad \Delta_{AB}\hat{w} \equiv \hat{w}_A - \hat{w}_B, \tag{15}$$

and solving (5)-(8) with (10), the DiD equations are (see Appendix 1):

$$\Delta_{12}\hat{p} = \frac{-\beta_R}{\varepsilon + \eta}\left\{\Delta_{AB}\hat{L} + \Delta\mathbf{J}\right\} \tag{16}$$

$$\Delta_{12}\hat{X} = \frac{\varepsilon\beta_R}{\varepsilon + \eta}\left\{\Delta_{AB}\hat{L} + \Delta\mathbf{J}\right\} \tag{17}$$

$$\Delta_{AB}\hat{w} = \beta_S\left(\frac{-\beta_R}{\varepsilon + \eta}\left\{\Delta_{AB}\hat{L} + \Delta\mathbf{J}\right\} + \Delta_{12}\hat{\chi}\right) \tag{18}$$

$$\Delta_{12}\hat{L}_f = \frac{(\varepsilon - \sigma)\beta_R}{\varepsilon + \eta}\left\{\Delta_{AB}\hat{L} + \Delta\mathbf{J}\right\} + (\sigma - 1)\left(\hat{\alpha}_{f1} - \hat{\alpha}_{f2}\right), \quad f = A, B. \tag{19}$$

Notation β_S, β_R, η, are respectively the Stolper-Samuelson, Rybczynski, and relative supply elasticities.[4] These elasticities capture the supply side responsiveness of wages and outputs with respect to prices or endowments (given technology) and take the form:

$$\beta_S = 1/(\omega_1 - \omega_2) > 1, \qquad \beta_R = 1/(\upsilon_A - \upsilon_B) > 1, \qquad \eta = \sigma(\beta_R\beta_S - 1) \geq 0. \tag{20}$$

As usual, β_S and β_R depend upon inter-sectoral factor intensity differences measured by cost share and employment share differences respectively. The relative supply elasticity, η, captures movements along the production possibility frontier when relative goods prices change. It is zero when $\sigma = 0$ (fixed coefficients), but when $\sigma > 0$ it rises with the responsiveness of relative outputs to endowments and relative wages to relative prices (β_R and β_S).

$\Delta\mathbf{J}$ is a parameter-weighted collection of TC terms which we refer to as the Jones Equivalent, in honour of Ron Jones. The Jones Equivalent is defined as:

$$\Delta\mathbf{J} \equiv \Delta_{AB}\hat{\lambda} + \sigma\big(\beta_S\Delta_{12}\hat{\chi} - \Delta_{AB}\hat{\lambda}\big), \tag{21}$$

where:

$$\Delta_{AB}\hat{\lambda} \equiv \hat{\lambda}_A - \hat{\lambda}_B, \quad \Delta_{12}\hat{\chi} \equiv \hat{\chi}_1 - \hat{\chi}_2. \tag{22}$$

Here $\Delta_{AB}\hat{\lambda}$ is the skilled-labour bias in the TC's factor savings, and $\Delta_{12}\hat{\chi}$ is the sector-1 bias in its cost savings.

3. Jones Equivalence

In his pioneering analysis of the HO model, Jones (1965) developed the powerful insight that technical change could be viewed as equivalent to a relative factor endowment change on the output side and a relative price change on the wage side. For instance, Jones (1965 p. 568) notes: "technological change, through its impact in reducing input coefficients, has precisely the same effects on the system as would a change in factor endowments". The insight, which we term Jones Equivalence, is useful in that it allows Rybczynski and Stolper-Samuelson intuition to be applied to the general equilibrium effects of TC. Jones (1965) did not provide a closed form solution linking the output and wage effects to technological parameters since he did not model TC explicitly.[5]

One of our contributions is to clarify the extent to which Jones Equivalence holds exactly. Any set of factor-augmenting TCs can be expressed in $\Delta\mathbf{J}$, and equivalence holds if their general equilibrium impact is the same as that of a corresponding change in relative factor endowments $\Delta_{AB}\hat{L} = \Delta\mathbf{J}$. Inspection of equations (16)-(19) indicates when equivalence holds, and gives the following proposition:

[4] These are supply-side relationships at given technology. The Stolper-Samuelson elasticity is $\beta_S \equiv \Delta_{AB}\hat{w}/\Delta_{12}\hat{p}$ (appendix 1, eq. A1.1). Rybczynski and relative supply elasticities come from the responses of supply to exogenous shifts in relative endowments and relative prices $\Delta_{12}\hat{X} = \beta_R\Delta_{AB}\hat{L} + \eta\Delta_{12}\hat{p}$ (appendix 1, eq. A1.3, at fixed technology).

[5] Using our notation, Jones introduces technical change into his model via the unit labour demand functions, $a_{fs} = a_{fs}[w_A/w_B, t]$ where a_{fs} is the unit labour-f input in sector s, and t is "the state of technology". Log differentiating, he expresses the proportional change in a_{fs} as $\hat{a}_{fs} = \hat{c}_{fs} + \hat{b}_{fs}$, where $\hat{c}_{fs}$ is the change in a_{fs} due to the relative wage change (i.e. $\Delta_{AB}\hat{w}$) and $\hat{b}_{fs}$ is "the measure of technological change that shows the alteration of $\hat{a}_{fs}$ that would take place at constant factor prices". His analysis explicitly links the $\hat{b}_{fs}$ to changes in key endogenous variables, thus implicitly linking them to the underlying change in technology.

Proposition 1: The Jones Equivalent of technical change.

The Jones Equivalent (JE) relative endowment change of a factor-augmenting TC is $\Delta_{\boldsymbol{AB}}\hat{L} = \Delta\mathbf{J}$. The general equilibrium impact of any factor-augmenting TC:

(i) On relative output and relative price is the same as that of its JE relative endowment change;

(ii) On the relative wage is the same as that of its JE relative endowment change plus the bias in the cost-savings effect, $\Delta_{12}\hat{\chi}$;

(iii)) On employment of each factor in each sector is the same as that of its JE relative endowment change if $\sigma = 1$. If $\sigma \neq 1$, there is an additional term containing the sector-bias in factor augmentation, namely $(\sigma - 1)\left(\hat{\alpha}_{f1} - \hat{\alpha}_{f2}\right)$.

Proof of Proposition 1 is by inspection of (16)-(19) noting that $\Delta\mathbf{J}$ and its JE relative endowment change, $\Delta_{AB}\hat{L}$, always enter additively, thereby having identical effects. In equations (16) and (17) all TC effects are contained in $\left\{\Delta_{AB}\hat{L} + \Delta\mathbf{J}\right\}$, while in expressions (18) and (19) further TC terms matter for the labour market impacts. These are the relative cost savings of the TC, $\Delta_{12}\hat{\chi}$, for the relative wage effect, and the sector-bias in factor augmentation of the TC, $(\sigma - 1)\left(\hat{\alpha}_{f1} - \hat{\alpha}_{f2}\right)$, for the jobs effects.

We discuss each of these effects – on price, output, wages and employment – in detail in the next sub-section. The Jones Equivalent is at the heart of our analytic framework, so we first discuss its components and build intuition for the mapping between particular forms of TC and $\Delta\mathbf{J}$.

As expressed in equation (21) $\Delta\mathbf{J}$ contains two parts. The first, $\Delta_{AB}\hat{\lambda}$, captures the direct changes in relative factor abundance measured in effective units; if there are no factor substitution possibilities, $\sigma = 0$, this drives the entire effect, including consequent changes in relative effective factor prices, $\left(\frac{w_A}{\alpha_{As}} / \frac{w_B}{\alpha_{Bs}}\right)$. If factor substitution is possible these factor price changes give rise to changes in factor-intensities in each sector, adding a second effect to the equivalence between TC and endowment change, and given by $\sigma\left(\beta_S \Delta_{12}\hat{\chi} - \Delta_{AB}\hat{\lambda}\right)$. So for any TC, $\Delta\mathbf{J}$ gives the relative change in factor endowment that would be equivalent to the TC, taking into account both the direct factor abundance effect and the changes in relative factor intensities due to the substitution effect (at given goods prices).

If TC takes the form of augmentation of just one of the factors in just one of the sectors, for example, $\hat{\alpha}_{B1} > 0$ with all other $\hat{\alpha}_{fs} = 0$, then the JE can be written $\Delta\mathbf{J} = [\upsilon_B(\sigma - 1) + \sigma\beta_S(1 - \omega_1)]\hat{\alpha}_{B1}$.[6] Notice that the term in square brackets switches from negative to positive as σ increases in the interval [0,1], changing the relative weights on factor abundance and factor substitution. This sign reversal points to the richness of possible results, as well as to the danger of working with an assumption such as Cobb-Douglas technology. In section 5 we give expressions for $\Delta\mathbf{J}$ with TC taking the restricted forms of sector-biased augmentation (but the same for both factors) and factor-biased (same in both sectors).

[6] The full form of $\Delta\mathbf{J}$ in terms of factor augmentation is: $\Delta\mathbf{J} = \left[\upsilon_A(1 - \sigma) + \sigma\beta_S\omega_1\right]\hat{\alpha}_{A1} + \left[(1 - \upsilon_A)(1 - \sigma) - \sigma\beta_S\omega_2\right]\hat{\alpha}_{A2} - \left[\upsilon_B(1 - \sigma) - \sigma\beta_S(1 - \omega_1)\right]\hat{\alpha}_{B1} - \left[(1 - \upsilon_B)(1 - \sigma) + \sigma\beta_S(1 - \omega_2)\right]\hat{\alpha}_{B2}$.

4. Analysing labour market impacts of TC with the Jones Equivalent framework

Expressions (16)-(19) provide a complete characterisation of how factor-augmenting TC influences relative prices, output, wages, and employment, and they illustrate how these impacts vary with underlying parameters associated with tastes, technology, and openness. More specifically, (4) and (20) show that the sign and magnitude of the relationships expressed in (16)-(19) vary: with the elasticity of demand (ε^o) and the size of the economy in which TC occurs ($\delta = [0, 1]$) via ε; with the factor substitution elasticity (σ) directly and via η; and with the intersectoral factor-intensity differences evaluated at the initial equilibrium, $\omega_1 - \omega_2$ and $\upsilon_A - \upsilon_B$, via β_S, β_R, and η. To provide a point of reference, we note that the framework is much simpler in the familiar small open economy case with fixed endowments. By definition, (16) is $\Delta_{12}\hat{p} = 0$ while (17)-(19) are $\Delta_{12}\hat{X} = \beta_R \Delta\mathbf{J}$, $\Delta_{AB}\hat{w} = \beta_S \Delta_{12}\hat{\chi}$, and $\Delta_{12}\hat{L}_f = \beta_R \Delta\mathbf{J} + (\sigma - 1)(\hat{\alpha}_{f1} - \hat{\alpha}_{f2})$ in this case.

To elucidate the analytic uses of the framework, we discuss the impact of TC on relative price, output, wages, and employment in sequence.

Price and output effects. Equations (16) and (17) show that any form of factor augmenting TC can be analysed as if it were an endowment change once its Jones equivalence, $\Delta\mathbf{J}$, is calculated. Any form of TC with $\Delta\mathbf{J} > 0$ will have the same impact on $\Delta_{12}\hat{p}$ as a rise in the relative endowment of skilled labour ($\Delta_{AB}\hat{L}$).

The equilibrium effect of $\Delta\mathbf{J} > 0$ (equivalently $\Delta_{AB}\hat{L} > 0$) on $\Delta_{12}\hat{p}$ and $\Delta_{12}\hat{X}$ involves the full range of general equilibrium mechanisms. When skilled labour becomes relatively more abundant, the Rybczynski theorem states that sector 1 will expand, but the increased abundance of good 1 will depress its relative price (when ε is finite) and this, via the Stolper-Samuelson theorem, will depress skilled labour's relative wage. Relatively cheaper skilled labour, however, will induce factor substitution in both sectors that absorbs some of the more abundant factor, and thus mitigates the Rybczynski effect.

Despite the complexity of the economic mechanisms involved, the sign of the relative price effect is unambiguously negative, and its size determined by a simple combination of parameters, namely $-\beta_R/(\varepsilon + \eta)$. The Rybczynski effect is apparent, and factor price changes and Stolper-Samuelson effects enter through the form of $\Delta\mathbf{J}$ and the presence of η in the denominator of (16). The denominator is the sum of the price responsiveness of demand and supply, $\varepsilon + \eta$, exactly analogous to demand and supply elasticities in a simple partial equilibrium model.

The associated change in output follows directly from the relative demand curve, $\Delta_{12}\hat{X} = -\varepsilon\Delta_{12}\hat{p}$. Thus, any form of TC with $\Delta\mathbf{J} > 0$ has a positive effect on sector-1's relative output, the magnitude governed by $\varepsilon\beta_R/(\varepsilon + \eta)$. Recalling that ε falls as the TC applies to a larger share of the world economy, as per (4), expression (17) shows that relative output expands more strongly when the countries experiencing the technical change have opportunities to trade with countries unaffected by the technical change.[7] The intuition is that the relative output change tends to be dampened by counteracting relative price changes. The dampening is stronger when the TC occurs widely, and the dampening is absent when it only occurs in a negligible portion of the world (small open economy case).

[7] See also Krugman (2000).

Relative wage effects. Relative wage effects do not satisfy Jones equivalence, as can be seen from (18). They can be understood by using (16) to rewrite (18) as $\Delta_{AB}\hat{w} = \beta_S(\Delta_{12}\hat{p} + \Delta_{12}\hat{\chi})$.[8] The first right-hand term is the price channel (which has Jones equivalence) while the second is the direct cost-saving channel operating through $\Delta_{12}\hat{\chi}$. Both the price and direct cost-saving channels impact wages via the Stolper-Samuelson elasticity, as expected. Intuitively, an increase in the relative supply of skilled labour (or TC that is Jones equivalent) tends to make skilled labour relatively less scarce and thus tends to depress its relative reward.

The price channel can be mitigated, amplified, or reversed by the direct cost-saving channel. When the TC favours cost savings in one sector ($\Delta_{12}\hat{\chi} \neq 0$), then the relative wage of the factor used intensively in the favoured sector will tend to rise. Intuitively, the direct cost channel works like the Stolper-Samuelson effect since a TC that lowers costs more in, say, sector 1 than sector 2 will push up the relative wage of the factor used intensively in sector 1. The point is that a higher relative price with no technical change is like a lower relative cost with no price change. For example, if the TC favours the skill-intensive sector ($\Delta_{12}\hat{\chi} > 0$), then the TC will tend to boost the relative productivity of skilled workers and thus their relative wage.

As far as we know, all existing results regarding the impact of factor augmenting TC on relative wages within the HO model are consolidated in equation (18). As an illustration of the consolidation, we note that the famous exchange between Leamer and Krugman on technology and wages (Leamer, 1998, 2000, Krugman, 2000) can be seen by inspection of (18). In a small open economy $\Delta_{AB}\hat{w} = \beta_S\Delta_{12}\hat{\chi}$, so the skill bias of technical change is irrelevant to wages, as Leamer claimed, while in a large open economy, (18) shows that both sector and factor bias matter as claimed by Krugman. Another example of consolidation concerns the well-known investigation of the links between TC and wages in the HO model by Xu (2001). That paper separately lists the signs of over 200 combinations of: i) the form of TC, ii) the taste and technology parameters, and iii) the economy's openness. All these are encompassed in (18).

Job creation/destruction effects. The impact of TC on the sector allocation of jobs of either type, $f = \mathrm{A, B}$, is captured fully by (19). Expression (19) can be separated into two channels: one, which we term the Jones Equivalent channel, $(\varepsilon - \sigma)\beta_R\Delta\mathbf{J}/(\varepsilon + \eta)$, is not factor-specific; the other, the sector-bias channel, $(\sigma - 1)(\hat{\alpha}_{f1} - \hat{\alpha}_{f2})$, is factor-specific.

The first channel, $(\varepsilon - \sigma)\beta_R\Delta\mathbf{J}/(\varepsilon + \eta)$, turns on the difference between output expansion, captured by ε, and factor substitution depending on σ. This is as in Hicks-Marshall laws of derived demand (Hicks 1932) and, in the context of TC, Graetz and Michaels (2018). Further insight can be derived by using (16) and (17), to write the expression as $\Delta_{12}\hat{X} + \sigma\Delta_{12}\hat{p}$. The output term is clear, simply moving both factors to the sector whose output has increased. The price effect works through relative wages and factor-substitution and will always have the opposite sign of the Rybczynski effects, since $\Delta_{12}\hat{X}$ and $\Delta_{12}\hat{p}$ have opposite signs. For example, if the TC is Jones equivalent to an expansion of the relative supply of skilled labour ($\Delta\mathbf{J} > 0$), then (17) shows that output will expand in the skill-intensive sector (via the Rybczynski mechanism). As a consequence the relative price of the skill-intensive sector will fall, (16). This will, via Stolper-Samuelson logic, tend to lower the relative wage of skilled workers and thus induce a substitution toward skilled workers in both sectors. This factor substitution works against the output effect since it raises the economy-wide usage of skilled workers relative to unskilled workers

[8] This also shows that Jones' observation that TC has the same effect as a change in relative goods prices is correct. However, since $\Delta_{12}\hat{p}$ is endogenously determined (except in the small open economy case) and depends on $\Delta\mathbf{J}$, there is not a one-to-one correspondence between the Jones Equivalent and relative wage effects.

and thus absorbs some of the rise in the relative abundance of effective units of skilled labour. The net effect depends on $\varepsilon - \sigma$ as shown in (19).

Adding the sector-bias channel, $\hat{\alpha}_{f1} - \hat{\alpha}_{f2} \neq 0$, has two effects. First, it has a direct impact on employment as it changes the stock of factor f measured in effective units. For instance, if the factor augmentation is biased towards sector 1 ($\hat{\alpha}_{f1} > \hat{\alpha}_{f2}$), then the shift in workers of type-f to sector 1 from sector 2 will be mitigated since fewer workers will be needed in sector 1 to produce equilibrium output. And second, it adds to the factor-substitution effects, through changing relative sector-specific wages measured in effective terms. The latter effects work very much as the factor-substitution effects discussed above; the only difference being that it is now sector-specific and may have different impact. If $\sigma = 1$, the two effects in the sector-bias channel exactly cancel each other; otherwise, there will be a net sector bias, depending on the sign of $\sigma - 1$.

It is noteworthy that the impacts of TC on wages and jobs, $\Delta_{AB}\widehat{w}$ and $\Delta_{12}\widehat{L}_f$, are not always positively correlated, so one cannot deduce the sign of employment effects from the sign of wage effects. For example, using (18), (19) and assuming that there is no sector bias, so $\hat{\alpha}_{f1} = \hat{\alpha}_{f2}$, the sign of $\Delta\mathbf{J}$ on the employment effect is given by $\varepsilon - \sigma$, while the sign of the relative wage effect depends on the balance between a negative price impact, $-\beta_R\Delta\mathbf{J}/(\varepsilon + \eta)$, and a positive relative cost effect, $\Delta_{12}\hat{\chi}$. Hence, the impact of TC on employment cannot be understood with reference to the wage effects. Employment effects must be considered separately.

5. Restricted TC: factor bias and sector bias

The Jones Equivalent approach simplifies general equilibrium analysis of TC by enabling the use of well-known theorems. It encompasses familiar forms of TC, such as skill-biased technical change, and to connect with this and other well-known special cases this section focuses on a restricted range of TC. Rather than considering the four TC parameters (the $\hat{\alpha}_{fs}$) independently, we restrict attention to factor-A biased TC (denoted as $\Delta_{AB}\Phi$ where Φ is a mnemonic for 'factor') and sector-1 biased TC (denoted as $\Delta_{12}\Sigma$ where Σ is a mnemonic for 'sector').

$$\Delta_{AB}\Phi = \hat{\alpha}_{A1} - \hat{\alpha}_{B1} = \hat{\alpha}_{A2} - \hat{\alpha}_{B2}, \qquad \Delta_{12}\Sigma = \hat{\alpha}_{A1} - \hat{\alpha}_{A2} = \hat{\alpha}_{B1} - \hat{\alpha}_{B2}. \tag{23}$$

In words, $\Delta_{AB}\Phi$ restricts the factor A versus factor B productivity progress to be identical across sectors while $\Delta_{12}\Sigma$ restricts the sector 1 versus sector 2 progress to be identical across factors.

These types of TC impose structure on our more general concepts. Imposing these restrictions, the skill-bias in factor savings is $\Delta_{AB}\hat{\lambda} = \Delta_{AB}\Phi + \Delta_{12}\Sigma/\beta_R$; the sector-1 bias in the cost savings is $\Delta_{12}\hat{\chi} = \Delta_{12}\Sigma + \Delta_{AB}\Phi/\beta_S$; and the sector bias in factor-f augmentation is $\hat{\alpha}_{f1} - \hat{\alpha}_{f2} = \Delta_{12}\Sigma$. Using these in the general Jones Equivalent, (21), and simplifying:

$$\Delta\mathbf{J} = \Delta_{AB}\Phi + \frac{(1+\eta)}{\beta_R}\Delta_{12}\Sigma. \tag{24}$$

This indicates that, if factor augmentation satisfies the restrictions in (23), then $\Delta\mathbf{J}$ can be expressed as this positive combination of factor and skill biased technical change. Pure factor-biased TC ($\Delta_{AB}\Phi \neq 0, \Delta_{12}\Sigma = 0$) is identical to $\Delta\mathbf{J}$ and thus, as per Proposition 1, affects relative price and output in a way that is identical to an increase in the relative supply of skilled labour. More generally a mixture of pure factor-biased and pure-sector biased TC combine linearly, allowing us to derive, for these restricted TCs, the DiD equations corresponding to (16)-(19):

$$\Delta_{12}\hat{p} = -\frac{\beta_R}{\varepsilon+\eta}\Delta_{AB}\Phi - \frac{1+\eta}{\varepsilon+\eta}\Delta_{12}\Sigma, \tag{25}$$

$$\Delta_{12}\hat{X} = \frac{\varepsilon\beta_R}{\varepsilon+\eta}\Delta_{AB}\Phi + \frac{\varepsilon(1+\eta)}{\varepsilon+\eta}\Delta_{12}\Sigma, \tag{26}$$

$$\Delta_{AB}\hat{w} = \frac{\varepsilon-\sigma+(\sigma-1)\beta_R\beta_S}{\varepsilon+\eta}\Delta_{AB}\Phi + \frac{\beta_S(\varepsilon-1)}{\varepsilon+\eta}\Delta_{12}\Sigma, \tag{27}$$

$$\Delta_{12}\hat{L}_A = \Delta_{12}\hat{L}_B = \frac{(\varepsilon-\sigma)\beta_R}{\varepsilon+\eta}\Delta_{AB}\Phi + \frac{(\varepsilon-1)(\sigma+\eta)}{\varepsilon+\eta}\Delta_{12}\Sigma. \tag{28}$$

Expressions (25)-(29) provide the closed-form solutions for the impact on prices, outputs, wages, and jobs in response to any combination of factor- and sector-biased TC satisfying (23). To boost intuition for the results and identify mechanisms, the impacts of pure factor- and pure sector-biased TC are considered in turn. We start with pure factor-biased TC, where $\Delta\mathbf{J} = \Delta_{AB}\Phi$.

Factor-biased TC. The most widely studied TC is skill-biased technical change (SBTC), so we start the analysis with pure factor-biased TC that favours skilled labour. To summarise, (25)-(28) imply:

Proposition 2: The general equilibrium impact of skill-biased TC.

The general equilibrium impacts of any pure SBTC ($\Delta_{AB}\Phi > 0, \Delta_{12}\Sigma = 0$) include:

(i) An increase in the relative output of the skill-intensive good, accompanied by a fall in its price if ε is finite.

(ii) The creation of jobs for both types of labour in the skill-intensive sector, and destruction of jobs in the unskilled-intensive sector if and only if $\varepsilon > \sigma$.

(iii) A rise in the relative wage of skilled workers if ε and σ are sufficiently large, a sufficient condition being $\varepsilon > 1$ and $\sigma \geq 1$.

(iv) The correlation between the employment and wage effects for skilled workers is positive if and only if $\{\varepsilon - \sigma\}$ and $\{\varepsilon - \sigma + (\sigma - 1)\beta_R\beta_S\}$ have the same sign. A sufficient condition is $\varepsilon > \sigma \geq 1$.

(v) If the share of the world experiencing TC is small then ε is relatively large (equation 4), hence it is more likely that conditions in ii, iii and iv are satisfied in which case SBTC both creates jobs in the skill-intensive sector and raises the relative wage of skilled workers.

Proof follows from inspection of (25)-(28). We discuss each part in turn.

The results on the goods-side are straightforward given Proposition 1.i, which tells us that the Jones Equivalent is exact for relative goods prices and outputs, and the fact that $\Delta\mathbf{J} = \Delta_{AB}\Phi$ for a pure factor-biased TC. The factor-side impacts are more involved.

Turning to the impact on the allocation of jobs, Proposition 2.ii and (28) shows that SBTC shifts jobs from the unskill-intensive sector (sector 2) to the skill-intensive sector (sector 1) as long as $\varepsilon > \sigma$. Again, this follows from Proposition 1, since there is no sector bias in this case. We think of $\varepsilon > \sigma$ as the intuitive case since (19) shows that an endowment change that makes skilled workers relatively more abundant ($\Delta_{AB}\hat{L} > 0$) leads to an expansion of skilled jobs in the skill-intensive sector only if $\varepsilon > \sigma$. An unexpected result for the jobs impact of SBTC is that the intersectoral jobs shift is identical for skilled and unskilled workers ($\Delta_{12}\hat{L}_A = \Delta_{12}\hat{L}_B$) despite the possibility of factor substitution. This result depends on our assumption that $\sigma_1 = \sigma_2 = \sigma$.

The relative wage result in Proposition 2.iii and (27) is less transparent. The condition for the numerator, $\varepsilon - \sigma + (\sigma - 1)\beta_R\beta_S$, to be positive can be written as $\sigma > (\beta_R\beta_S - \varepsilon)/(\beta_R\beta_S - 1)$. Recalling that $\beta_R\beta_S > 1$ makes clear the sufficient condition in 2.iii. Intuition is as in the discussion of the relative wage effect in Section 4, with price channel and direct cost-saving channel operating.

Proposition 2.iv, illustrates the important point that wage and employment effects can go in opposite directions, even with this restricted form of TC. For example, if $\varepsilon = 1$ then the numerator of (27) is $(1 - \sigma)(1 - \beta_R\beta_S)$ while that of (28) is $(1 - \sigma)$, two expressions with opposite signs. Thus, if σ >1, the relative wage of skilled workers will go up, while the employment of both factors in the skill-intensive sector declines. The intuition for this result is that even if employment in the A-intensive sector declines, the substitution effect towards more use of skilled labour in both sectors is so strong that the overall relative demand for factor A increases.

The final part, 2.v, links back to the importance of whether the region experiencing the TC is large or small relative to the rest of the world, and hence the prospects for sales expansion to areas not experiencing the TC. If the region is relatively small then employment expansion in the skill-intensive sector and relative wage growth for skilled workers is more likely.

Sector-biased TC. Sector-biased TC occurs when, for example, TC takes place in manufacturing but not services. Formally, pure sector-biased TC involves $\Delta_{AB}\Phi = 0,\ \Delta_{12}\Sigma \neq 0$. Effects on relative price, output, wage and employment are given by (25)-(28). As it turns out, the impacts of a pure sector-1 biased TC ($\Delta_{AB}\Phi = 0, \Delta_{12}\Sigma > 0$) are simpler than those of SBTC. From (25) and (26), such TC increases the relative supply and reduces the relative price of the skill-intensive good as long as ε is finite. Sector-1 biased TC also increases the relative wage of skilled workers and shifts jobs to sector 1 as long as the relative price is sufficiently unresponsive to relative output changes, specifically $\varepsilon > 1$. This also means that the wage and jobs impacts are always positively correlated for pure sector-biased TCs.

To summarise:

Proposition 3: The general equilibrium impact of sector-biased TC.

The general equilibrium impact of any pure sector-biased TC ($\Delta_{AB}\Phi = 0, \Delta_{12}\Sigma > 0$) that favours sector-1 include:

(i) An increase in the relative output of the skill-intensive good, accompanied by a fall in its price if ε is finite.

(ii) The creation of jobs in the skill-intensive sector and destruction of jobs in the unskilled-intensive sector if and only if $\varepsilon > 1$.

(iii) The rise in the relative wage of skilled workers if and only if $\varepsilon > 1$;

(iv) A positive correlation between relative wage and employment effects.

Proof is by inspection of (25)-(28) noting that $\varepsilon, \sigma, \beta_S, \beta_R$, and η are non-negative. As in the SBTC case, the smaller the share of the economy experiencing TC, the more likely is it that a sector-1 biased TC implies increased relative wage of skilled workers and increased employment of both factors in the skill-intensive sector.

6. Tradable and non-tradable sectors

To our best knowledge, all theoretical studies of TC in an open economy follow the standard trade theory practice of assuming all goods are traded. Non-traded sectors, however, account for a large share of advanced economies' output and employment. Many of the workers who lost jobs in manufacturing due to automation and globalisation found jobs in low-skill, non-traded sectors such as hospitality, retail, and transport sectors. This raises questions: How does the presence of a non-traded sector change the impact of TC on jobs and wages? Does the possibility of employment in the non-tradable sector dampen the impacts by providing a separate margin of adjustment to TC?

In this section, we capture this extra margin of adjustment by adding a non-tradeable sector to the model. The behaviour of the HO model with a non-traded sector has been studied (e.g. Ethier 1972, Helpman and Krugman 1985, Xu 2003, and Jones 2012) but such models have not, to our knowledge, been utilised to investigate the labour market impacts of TC, so we keep the extension as simple as possible. Goods 1 and 2 are tradable, as before, and there is a further sector, N, which produces non-tradable output. We assume that this sector does not experience technical progress, has fixed labour input coefficients, and that preferences between tradeable and non-tradeable goods are Cobb-Douglas.[9] We continue to assume regularity conditions that ensure the country has positive production of both traded goods, so (1), (2) and (4) continue to hold for traded goods.[10] For clarity's sake, we assume that factor endowments do not change, although Jones Equivalence allows us to discuss biased changes in technology as if they were changes in the relative endowment (subject to the provisos in Proposition 1).

The N-sector interacts with the supply side of the economy by using some of the economy's endowment of factors of production, implying that the factor supplies available to the tradables sectors are residual to what is used in non-traded production. Non-traded production, and thus employment, are determined by the demand for non-traded output. Given Cobb-Douglas preferences, spending on non-tradables is $p^N X^N = s^N Y$ where Y is total income, X^N is sector-N demand, and s^N is the constant expenditure share on non-tradables. Log differentiating, the equation of change for non-tradable demand, and hence production, is $\hat{X}^N = \hat{Y} - \hat{p}^N$. With fixed input coefficients, $\hat{X}^N$ determines N-sector employment, so:

$$\hat{L}_A^N = \hat{L}_B^N = \hat{X}^N = \hat{Y} - \hat{p}^N, \tag{29}$$

where L_f^N is N-sector employment of labour of type $f = A, B$.

Expression (29) is the linkage between the N-sector and the rest of the economy. Changes in N-sector demand, and hence output and employment, draw workers out of, or release workers to, the tradable sectors, so the N-sector provides an additional margin of adjustment for any TC. Allowing factor substitution and/or TC in the non-traded sector would make (29) more complex without altering the basic mechanism. Given the lack of TC in sector N, p^N depends solely on factor prices, as does income $Y = w_A L_A + w_B L_B$.[11] Changes in each are respectively $\hat{p}^N = \omega^N \hat{w}_A + (1 - \omega^N)\hat{w}_B$ and $\hat{Y} = \overline{\omega}\hat{w}_A + (1 - \overline{\omega})\hat{w}_B$, where ω^N and $\overline{\omega}$ are the factor-A cost shares in X^N and income shares in Y. Using these expressions in (29), the change in $\hat{X}^N$ is given by

$$\hat{X}^N = (\overline{\omega} - \omega^N)\Delta_{AB}\hat{w}. \tag{30}$$

[9] The demand elasticity between good 1 and good 2 remains ε^o. This could come from a 2-level CES utility function, with elasticity 1 between good-N and a composite of the two traded goods at the top level, and elasticity ε^o between traded goods 1 and 2 at the second level.

[10] Implying that factor prices are determined from (1), and not directly affected by the non-traded sector.

[11] As long as factor endowments are given.

For example, if non-tradable output is skill intensive relative to the economy as a whole ($\omega^N > \bar{\omega}$) a rise in the relative wage of skilled workers will boost p^N more than Y so X^N will fall, this releasing skilled and unskilled workers from the N-sector into the tradable sectors.

The next step in the analysis is to show how flows of labour into or out of the N-sector affect the relative availability of labour to the tradeable sectors. Defining the residual supply of labour to the traded sectors as $L_f^T = L_f - L_f^N, f = A, B$, and the bias in the movement of workers into tradeable sectors as $\Delta_{AB}\hat{L}^T = \hat{L}_A^T - \hat{L}_B^T$, we can write (see Appendix 2):

$$\Delta_{AB}\hat{L}^T = \frac{s^N(\omega^T - \omega^N)}{(1 - s^N)\omega^T(1 - \omega^T)}\hat{X}^N. \tag{31}$$

The expression follows from manipulations of factor-intensity definitions, using $\bar{\omega} = (1 - s^N)\omega^T + s^N\omega^N$, where ω^T is the average factor-A cost share in the traded sectors (Appendix 2). Combining (30) and (31) gives us the key mechanism that arises with inclusion of a non-traded sector.

$$\Delta_{AB}\hat{L}^T = \varphi\Delta_{AB}\hat{w}, \text{ where } \varphi \equiv \frac{s^N(\omega^T - \omega^N)^2}{\omega^T(1 - \omega^T)} \geq 0. \tag{32}$$

The parameter φ captures the fact that a change in relative wages changes the relative supply of factors to tradable sectors, the effect coming from both the price and income effects of wage changes on N-sector demand and output. The strength of the mechanism, φ, increases with the square of the difference between the factor intensity of the N-sector and other sectors.[12] For example, if $\omega^T = \omega^N$, the labour flows in or out of the N-sector would not change the relative abundance of factors in the rest of the economy and would thus not change relative prices, wages, or outputs. However, for any $\omega^T \neq \omega^N$, an increase in the relative wage, $\Delta_{AB}\hat{w} > 0$, will increase the relative supply of skilled labour to the traded sectors. If $\omega^T > \omega^N$, a TC that raises the relative wage of skilled labour pulls workers out of the tradable sectors and into the N-sector to satisfy the higher demand for N-goods, as (30) and (31) show. Since the N-sector in this case is intensive in unskilled labour, the result is an increase in the *relative* supply of skilled labour to the traded sectors. If, on the other hand, $\omega^T < \omega^N$, the same TC would result in the N-sector shrinking and releasing labour to the traded sectors. In this case more skilled than unskilled labour is released, thus again increasing the *relative* supply of skilled workers to the traded sectors. The strength of parameter φ is proportional to the importance of the N-sector in the economy as measured by s^N. The baseline model in section 2 is a special case of the model in this section since $\varphi = 0$ when $s^N = 0$.

To work through the full implications of this mechanism we note first that, absent a change in the factor endowment of the total economy, the change $\Delta_{AB}\hat{L}$ in the two-sector economy (equations (16)-(19)) is now equal to $\Delta_{AB}\hat{L}^T$. Using $\Delta_{AB}\hat{L}^T$ from (32) in the role of $\Delta_{AB}\hat{L}$we can write:

$$\Delta_{AB}\hat{w}\left\{1 + \frac{\varphi\beta_S\beta_R}{\varepsilon + \eta}\right\} = \beta_S\left\{\frac{-\beta_R}{\varepsilon + \eta}\Delta\mathbf{J} + \Delta_{12}\hat{\chi}\right\}. \tag{33}$$

Since the bracketed term on the left-hand side of this equation weakly exceeds unity, and the right-hand side is the same as in (18) with given factor endowments, the effect of technical change on relative wages is dampened, and the more so the greater is φ. It is in this sense that labour flows into or out of

[12] This follows from the fact that $(\omega^T - \omega^N)$ determines the correlation between $\hat{X}^N$ and $\Delta_{AB}\hat{w}$ in (30), as well as the correlation between $\Delta_{AB}\hat{L}^T$ and $\hat{X}^N$ in (31).

the N-sector have a dampening effect. The ultimate rise in $\Delta_{AB}\widehat{w}$ induced by TC will be smaller in a model with an N-sector compared to that without a non-traded sector.

In summary, regardless of the N-sector's factor intensity relative to the traded sectors (as long as $\omega^N \neq \omega^T$), a TC-induced increase in skilled wages, $\Delta_{AB}\widehat{w} > 0$, stimulates differential labour flows that make skilled labour relatively more abundant in the tradable sectors and thereby dampens the wage effects compared to what would have happened without an N-sector.

To derive the full range of results, use $\Delta_{AB}\widehat{w} = \Delta_{AB}\widehat{L}^T/\varphi$ again to replace $\Delta_{AB}\widehat{w}$ in (33). Defining $\eta' \equiv \eta + \varphi\beta_S\beta_R > \eta$ and rearranging gives:

$$\left\{\Delta_{AB}\widehat{L}^T + \Delta\mathbf{J}\right\} = \frac{\varepsilon + \eta}{\varepsilon + \eta'}(\Delta\mathbf{J} + \varphi\beta_S\Delta_{12}\hat{\chi}). \tag{34}$$

The left-hand side of this expression is the Jones Equivalent shock to the two sector (traded goods) economy – the expression that enters equations (16) – (19). The induced change $\Delta_{AB}\widehat{L}^T$ is captured by the additional terms on the right-hand side, so the effect of TC on prices, output, wages, and employment are found by using (34) in (16) – (19). Thus, the effect of TC contained in $\Delta\mathbf{J}$ is dampened in all cases, since $\eta' > \eta$. However, TC also enters through additional term $\varphi\beta_S\Delta_{12}\hat{\chi}$ which, if $\Delta\mathbf{J}$ and $\Delta_{12}\hat{\chi}$ have different signs, may dampen, amplify, or even change the sign of the impact on prices, output and employment. For example, if $\Delta\mathbf{J} > 0$ and there is a cost-advantage to sector 2, so $\Delta_{12}\hat{\chi} < 0$, then the existence of a non-traded sector may switch the sign of $\Delta_{12}\hat{p}$ from negative to positive, and also change the signs of the production and employment effects.[13]

For the restricted range of TC considered in Section 5, the three-sector DiD equations are:

$$\Delta_{12}\hat{p} = -\frac{\beta_R(1+\varphi)}{\varepsilon + \eta'}\Delta_{AB}\Phi - \frac{(1+\eta')}{\varepsilon + \eta'}\Delta_{12}\Sigma \tag{35}$$

$$\Delta_{12}\widehat{X} = \frac{\varepsilon\beta_R(1+\varphi)}{\varepsilon + \eta'}\Delta_{AB}\Phi + \frac{\varepsilon(1+\eta')}{\varepsilon + \eta'}\Delta_{12}\Sigma \tag{36}$$

$$\Delta_{AB}\widehat{w} = \frac{\epsilon - \sigma + (\sigma - 1)\beta_R\beta_S}{\varepsilon + \eta'}\Delta_{AB}\Phi + \frac{\beta_S(\varepsilon - 1)}{\varepsilon + \eta'}\Delta_{12}\Sigma \tag{37}$$

$$\Delta_{12}\widehat{L}_A = \Delta_{12}\widehat{L}_B = \frac{(\varepsilon - \sigma)\beta_R(1+\varphi)}{\varepsilon + \eta'}\Delta_{AB}\Phi + \frac{(\varepsilon - 1)(\sigma + \eta')}{\varepsilon + \eta'}\Delta_{12}\Sigma. \tag{38}$$

These differ from (25)-(28) due to the presence of φ and the replacement of η by $\eta' > \eta$. The general dampening effect of the N-sector's inclusion shows up in the fact that $\varepsilon + \eta$ is replaced by $\varepsilon + \eta'$ in the denominator of each of the expressions (35)-(38). In the relative wage equation, (37), this is the only difference so, as noted for the general TC case, the relative wage impact of TC is always muted.

Other coefficients in these equations have changes in the numerator as well as the denominator. For a pure SBTC ($\Delta_{AB}\Phi > 0, \Delta_{12}\Sigma = 0$) net effects on relative prices, output, and labour allocation are amplified if and only if $(\varepsilon - \sigma) + \beta_S\beta_R(\sigma - 1) > 0$, since this condition means that $(1+\varphi)/(\varepsilon + \eta')$ (the coefficients on $\Delta_{AB}\Phi$ in these equations) are increasing in φ.[14]

[13] A very simple example could be $\hat{\alpha}_{A2} > 0$ and all other $\hat{\alpha}_{fs} = 0$. Then, $\Delta_{12}\hat{\chi} < 0$, and depending on parameters, we may well have $\Delta\mathbf{J} > 0$ while $\Delta\mathbf{J} + \varphi\beta_S\Delta_{12}\hat{\chi} < 0$; i.e., the existence of a non-traded sector would switch the sign of the (generalised) Jones Equivalence.

[14] Including its effect on η'. Obtaining by inspection of the term, using $\eta' = \sigma(\beta_R\beta_S - 1) + \varphi\beta_S\beta_R$.

Considering pure sector-biased TC ($\Delta_{AB}\Phi = 0, \Delta_{12}\Sigma \neq 0$), the impacts on relative price and output are amplified if $\varepsilon > 1$, this implying $(1+\eta')/(\varepsilon+\eta')$ is increasing with φ . The impact on the division of employment in the tradable sectors is amplified if $\varepsilon > \sigma$, as this ensures that $(\sigma+\eta')/(\varepsilon+\eta') > (\sigma+\eta)/(\varepsilon+\eta)$, in equation (38).

In both these cases the conditions for the N-sector to amplify the price and output effects are the same as the conditions for $\Delta_{AB}\widehat{w} > 0$. The logic for this is the following: we know from (33) that $\varphi > 0$ always dampens the relative wage effects, and since we can write $\Delta_{AB}\widehat{w} = \beta_S\{\Delta_{12}\hat{p} + \Delta_{12}\hat{\chi}\}$, where $\Delta_{12}\hat{\chi}$ is not affected by φ, the dampening effect must come from $\Delta_{12}\hat{p}$. Hence, if $\Delta_{AB}\widehat{w} > 0$ the dampening effect comes from an amplified (negative) $\Delta_{12}\hat{p}$, whereas, if $\Delta_{AB}\widehat{w} < 0$, it comes from a dampened (negative) $\Delta_{12}\hat{p}$.

We note two further points. First, as before, wage and employment effects always have the same sign for pure sector-biased TC, but may have opposite signs if there is factor-biased TC. The condition for this to occur is modified by the presence of φ in (38) but, as before, a sufficient condition for positive correlation between wage and employment impacts is that $\varepsilon > \sigma \geq 1$.

Second, the presence of the N-sector never switches the sign of any of the effects of pure factor- or sector-biased TC, although if these occur in combination and with different signs, then this need not be true – signs may change (except for the relative wage effect). For example, suppose that that $\Delta_{AB}\Phi$ and $\Delta_{12}\Sigma$ have different signs, and their combination is such that – absent the N-sector – $\Delta_{12}\hat{p}$ is close to zero (either positive or negative). Raising φ changes the coefficients on the TCs in a manner that is not proportional, creating the possibility of sign changes in their combined effect on prices, output and employment, although not on relative wages.[15] To summarise:

Proposition 4: Impact of TC with non-tradables.

If the N-sector has different factor intensity from the rest of the economy:

(i) the magnitude of any TC-induced change in relative wages is dampened by the presence of the N-sector, but its sign is unchanged. Dampening is greater the larger the N-sector.

(ii) the presence of a non-traded sector can change the sign of the relevant (generalised) Jones Equivalence, and hence change the sign of the price, output and employment effects.

In the special case of where TC is restricted to factor-biased and sector-biased:

(iii) The magnitude of the impacts on relative price, output, and employment in the traded goods sectors can be amplified, or dampened depending on elasticities.

(iv) The signs of the impacts of TC that is either purely factor-biased or purely sector-biased are never switched by the presence of an N-sector; however, the signs can be switched if both forms of biased TC are present and they have opposite signs.

The proof is by inspection of (35)-(38).

[15] As a specific example, assume $\Delta_{AB}\Phi > 0$, $\Delta_{12}\Sigma < 0$, and that the two effects cancel each other in the model without an N-sector (equation (25)), so the price effect is zero. Comparing the coefficients in (35) and (25), it is clear that the relative impact of $\varphi > 0$ on the coefficient for $\Delta_{AB}\Phi$ is greater than on the coefficient for $\Delta_{12}\Sigma$ if $(1+\varphi) > \frac{(1+\eta')}{(1+\eta)}$. This is satisfied if $1+\eta > \beta_S\beta_R$, which again, using the definition of η, requires $\sigma < 1$. Hence, in this case, the existence of a non-traded sector ($\varphi > 0$) would change the relative price effect, $\Delta_{12}\hat{p}$, from zero to negative if $\sigma < 1$ and from zero to positive if $\sigma > 1$.

7. Concluding remarks

Evidence points to a skill-bias in the TC experienced by advanced economies over the last few decades. There are some signs, however, that TC in coming decades will be quite different. The emergence of highly effective and easy to use AI-trained software models may help less skilled workers more than skilled workers. Machine learning, based on massive datasets, has provided computers with a new range of human-like cognitive skills, some of which are substitutes for jobs performed by highly skilled workers. As such, it may prove to be unskilled-biased TC rather than SBTC, or it could lead to productivity progress for a single factor in some sectors of the economy, but not in all.

What will be the impact of this new TC on labour markets in open economies? How will the TC's impact be influenced by openness and the portion of the world economy experiencing the TC? This paper provides a compact and intuitive framework that consolidates existing results in the literature, and allows exploration of the effects of different combinations of sector and skill specific TC. The framework is used to extend the theoretical study of trade-technology-labour linkages to include explicit results on the size and direction of TC's impact on the sector allocation of employment. Among other things, we make explicit what the Jones Equivalent (of TC and labour endowment changes) looks like and when it applies perfectly, and we show that for labour market effects, additional parameters matter. Furthermore, we illustrate the need to analyse both relative wage and relative employment effects, as these are not always correlated. We extend the analysis to cover a non-traded sector, finding that the extension can change the magnitude of impacts (on prices, outputs, wages, and employment), and can change the signs of the impacts on price, output and labour market outcomes for certain forms of TC. However, the existence of a non-traded sector will always dampen the relative wage effect of any TC, while keeping the sign unchanged.

While technical change and globalisation have been forceful influences on economic outcomes around the world, climate change may lead to massive international migration (Cruz and Rossi-Hansberg, 2021). Since our framework is based on the Jones' inspired equivalence between labour endowments changes and technical changes, our framework can be used to study the impact of exogenous migration shocks on relative price, output, wage, and employment.

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Appendix 1: The baseline model

In this appendix we provide a complete derivation and discussion of the key equations of the Section 2 model in the context of assuming CES technology. The key equations (16)-(19) are more general, but as long as the elasticity of substitution is assumed to be the same and constant for both sectors, it coincides with the equilibrium conditions developed from CES.

Derivation of (16)-(19).

Supply side: The pricing conditions, (1), with CES unit cost functions are:

$$p_s = c_s = \left[\Omega_s \left(\frac{w_A}{\alpha_{As}}\right)^{1-\sigma} + (1-\Omega_s)\left(\frac{w_B}{\alpha_{Bs}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \qquad s = 1,2.$$

Log differentiation yields (5). To build intuition for wage effects, we rewrite (5) as:

$$\omega_s \hat{w}_A + (1-\omega_s)\hat{w}_B = \hat{p}_s + \hat{\chi}_s, \qquad s = 1,2,$$

and solve the pair of equations, using (20), to get:

$$\Delta_{AB}\hat{w} = \beta_S\{\Delta_{12}\hat{p} + \Delta_{12}\hat{\chi}\}. \qquad (A1.1)$$

This illustrates a form of Jones Equivalence in that the sectoral cost savings, $\hat{\chi}_s$, act on relative wages in a manner that is isomorphic to an exogenous price change via the Stolper-Samuelson theorem.

Differentiating the cost function with respect to wages yields the labour demand functions:

$$L_{fs} = X_s\frac{\partial c_s}{\partial w_f} = \Omega_s\frac{X_s}{\alpha_{fs}}\left(\frac{p_s}{w_f/\alpha_{fs}}\right)^{\sigma}, \qquad s = 1,2, f = A, B.$$

Log differentiating the labour demand functions:

$$\hat{L}_{fs} = \hat{X}_s + \sigma(\hat{p}_s - \hat{w}_f) + (\sigma - 1)\hat{a}_{fs}, \qquad s = 1,2, f = A, B. \qquad (A1.2)$$

Using this in (7), with the definitions in (9) and (13), and solving for the output changes yields a pair of equations that define $\hat{X}_1$ and $\hat{X}_2$:

$$\begin{bmatrix}\hat{X}_1\\ \hat{X}_2\end{bmatrix} = \beta_R\begin{bmatrix}(1-\upsilon_B) & -(1-\upsilon_A)\\ -\upsilon_B & \upsilon_A\end{bmatrix}\begin{bmatrix}\hat{L}_A + (1-\sigma)\,\hat{\lambda}_A - \sigma[\upsilon_A\hat{p}_1 + (1-\upsilon_A)\hat{p}_2 - \hat{w}_A]\\ \hat{L}_B + (1-\sigma)\,\hat{\lambda}_B - \sigma[\upsilon_B\hat{p}_1 + (1-\upsilon_B)\hat{p}_2 - \hat{w}_B]\end{bmatrix}.$$

Taking the difference, and using (A1.1), we get:

$$\Delta_{12}\hat{X} = \beta_R\{\Delta_{AB}\hat{L} + (1-\sigma)\,\Delta_{AB}\hat{\lambda} + \sigma\beta_S\Delta_{12}\hat{\chi}\} + \sigma(\beta_R\beta_S - 1)\Delta_{12}\hat{p}.$$

Gathering terms into $\Delta\mathbf{J}$, the collection of parameters defined in (21), and η from (20), we get:

$$\Delta_{12}\hat{X} = \beta_R(\Delta_{AB}\hat{L} + \Delta\mathbf{J}) + \eta\Delta_{12}\hat{p}. \qquad (A1.3)$$

Note that $\eta = \sigma(\beta_R\beta_S - 1)$ is non-negative since $\sigma \geq 0$, and, from (20), β_R and β_S always have the same sign and both have absolute values in excess of 1. Expression (A1.3) shows why we call η the general equilibrium supply elasticity. In absence of TC and factor shocks (i.e., when $\Delta_{AB}\hat{L} + \Delta\mathbf{J} = 0$), (A1.3) shows that η equals $\Delta_{12}\hat{X}/\Delta_{12}\hat{p}$ (response of relative output to relative price). For example, the elasticity of relative output to an exogenous relative price shock is equal to η.

Employment effects derive from (A1.2). Using the definition in (14) to express the jobs impact in DiD notation, and (A1.2), we get:

$$\Delta_{12}\hat{L}_f = \Delta_{12}\hat{X} + \sigma\Delta_{12}\hat{p} + (\sigma - 1)(\hat{\alpha}_{f1} - \hat{\alpha}_{f2}). \qquad (A1.4)$$

Using (8) to eliminate $\Delta_{12}\hat{X}$ yields $\Delta_{12}\hat{L}_f = (\varepsilon - \sigma)(-\Delta_{12}\hat{p}) + (\sigma - 1)(\hat{\alpha}_{f1} - \hat{\alpha}_{f2})$. Equation (19) follows from this where (16) is used to eliminate $\Delta_{12}\hat{p}$. That completes the derivation of (16)-(19).

Demand side and equilibrium: Writing (8) as $\Delta_{12}\hat{p} = (-1/\varepsilon)\Delta_{12}\hat{X}$, substituting it into (A1.3), and simplifying, we get (17). Substituting (17) back into (8), with the definitions in (15), yields (16). Using (16) in (A1.1) gives (18).

Derivation of $\boldsymbol{\varepsilon}$. Derivation of $\varepsilon = \left((1-\delta)\,\eta^* + \varepsilon^o\right)/\delta$ in (4). Sector s output of home is X_s and that of the rest of the world is X_s^*. Consumers are identical worldwide, so world relative demand is $D(p_1/p_2)^{-\varepsilon^0}$, and rest of the world relative supply is given by $X_1^*/X_2^* = S(p_1/p_2)^{\eta^*}$, with η^* the rest-of-world supply elasticity. The world market clearing condition is $(X_1 + X_1^*)/(X_2 + X_2^*) = D(p_1/p_2)^{-\varepsilon^0}$ so, differentiating,

$$\frac{d(X_1 + X_1^*)}{(X_1 + X_1^*)} - \frac{d(X_2 + X_2^*)}{(X_2 + X_2^*)} = -\varepsilon^0(\hat{p}_1 - \hat{p}_2).$$

Proportionate changes in world supply of each good are $\frac{d(X_s + X_s^*)}{(X_s + X_s^*)} = \delta_s\hat{X}_s + (1-\delta_s)\hat{X}_s^*$, $s = 1, 2$, where δ_s is the shares of home in production of good s. Equality of changes in world supply and demand is therefore

$$\delta_1\hat{X}_1 + (1-\delta_1)\hat{X}_1^* - \delta_2\hat{X}_2 - (1-\delta_2)\hat{X}_2^* = -\varepsilon^0(\hat{p}_1 - \hat{p}_2)\,.$$

If $\delta_1 = \delta_2 = \delta$ then this expression can be rearranged, and using the rest of the world supply response, $\hat{X}_1^* - \hat{X}_2^* = \eta^*(\hat{p}_1 - \hat{p}_2)$, gives

$$\delta(\hat{X}_1 - \hat{X}_2) + (1-\delta)\,\eta^*(\hat{p}_1 - \hat{p}_2) = -\varepsilon^0(\hat{p}_1 - \hat{p}_2), \qquad (A1.5)$$

Gathering terms, we see that the responsiveness of relative world market prices, $\Delta_{12}\hat{p} = \hat{p}_1 - \hat{p}_2$, to relative home output, $\Delta_{12}\hat{X} = \hat{X}_1 - \hat{X}_2$, is $-(1/\varepsilon)$, where $\varepsilon = \left((1-\delta)\,\eta^* + \varepsilon^0\right)/\delta$. This demonstrates the claim in the text that ε links to the structural demand parameter as the TC is applied to larger shares of the global economy, with $\varepsilon = \varepsilon^0$ for $\delta = 1$, and $\varepsilon = \infty$ for $\delta = 0$ (small open economy case).

Appendix 2: Non-traded sector

The appendix provides a more detailed derivation of the model with a non-traded sector. Due to the Cobb-Douglas assumption, expenditure on non-tradables is $p^N X^N = s^N Y$ where p^N and X^N are the price and output of the N-sector, Y is economy-wide income and s^N is the constant expenditure share. Log differentiating, $\hat{X}^N = \hat{Y} - \hat{p}^N$. With perfect competition and constant returns, $p^N = w_A L_A^N + w_B L_B^N$, where L_f^N is employment of factor f in the N-sector, and $Y = w_A L_A + w_B L_B$, so changes in $\hat{p}^N$ and $\hat{Y}$ are linked to wage changes by:

$$\hat{p}^N = \omega^N\hat{w}_A + (1-\omega^N)\hat{w}_B, \quad \hat{Y} = \bar{\omega}\hat{w}_A + (1-\bar{\omega})\hat{w}_B,$$

where $\omega^N = w_A L_A^N/p^N$ is the cost share of factor-A in sector N and $\bar{\omega} = w_A L_A/Y$ is factor-A's cost/income share in the economy as a whole. As s^N is constant, $\bar{\omega} = (1 - s^N)\omega^T + s^N\omega^N$, where ω^T is factor-A's cost share in the tradeable sectors:

$$\omega^T = \frac{w_A(L_A - L_A^N)}{(1 - s^N)Y}.$$

Here $(1 - \mathrm{s}^{\mathrm{N}})Y$ is the spending on, and thus cost of, tradeable goods, and $w_A(L_A - L_A^N)$ is the payments to, and thus cost of factor-A in the tradable sectors.

$\hat{X}^N$ is linked to wage changes, since $\hat{X}^N = \hat{Y} - \hat{p}^N$ can be written, using the above expressions, as $\hat{X}^N = \bar{\omega}\hat{w}_A + (1 - \bar{\omega})\hat{w}_B - [\omega^N \hat{w}_A + (1 - \omega^N)\hat{w}_B]$, which yields (30). Since $\bar{\omega} - \omega^N$ simplifies to $(1 - s^N)(\omega^T - \omega^N)$, it follows that:

$$\hat{X}^N = (1 - s^N)(\omega^T - \omega^N)\Delta_{AB}\hat{w}. \qquad (A2.1)$$

This establishes the relationship between non-traded production and the change in relative wages.

Turning to employment, the labour supply to the traded sectors is a residual as all non-traded production must be undertaken with domestic labour, namely $dL_f^T = dL_f - dL_f^N$ which gives:

$$\hat{L}_f^T = \frac{\hat{L}_f L_f}{L_f^T} - \frac{\hat{L}_f^N L_f^N}{L_f^T}.$$

Assuming that the overall factor endowments are given, $\hat{L}_f = 0$ (for both factors) and using the assumption of fixed labour-input coefficients in the non-traded sector, we have $\hat{L}_A^N = \hat{L}_B^N = \hat{X}_N$. Using these expressions, and defining the factor-bias in the movement of factor-A versus factor-B to the traded sectors from the non-traded sector as $\Delta_{AB}\hat{L}^T = \hat{L}_A^T - \hat{L}_B^T$, we have, $\Delta_{AB}\hat{L}^T = -\left(\frac{L_A^N}{L_A^T} - \frac{L_B^N}{L_B^T}\right)\hat{X}^N$, which can be written as:

$$\Delta_{AB}\hat{L}^T = \frac{s^N(\omega^T - \omega^N)}{(1 - s^N)\omega^T(1 - \omega^T)}\hat{X}^N,$$

since $\frac{L_A^N}{L_A^T} = \frac{s^N \omega^N}{(1 - s^N)\omega^T}$ and $\frac{L_B^N}{L_B^T} = \frac{s^N(1 - \omega^N)}{(1 - s^N)(1 - \omega^T)}$.

Using (A2.1) in this, yields (32) in the text, namely:

$$\Delta_{AB}\hat{L}^T = \varphi\Delta_{AB}\hat{w}, \qquad \varphi \equiv \frac{s^N(\omega^T - \omega^N)^2}{\omega^T(1 - \omega^T)} \geq 0, \qquad (A2.2)$$

where $\varphi > 0$ as long as $\omega^T \neq \omega^N$ and $s^N > 0$.

The next step involves the realisation that, $\Delta_{AB}\hat{L}^T$, which is the factor-bias in the movement of factor-A versus factor-B to the traded sectors from the non-traded sector, plays the same role in this three-sector economy as the factor-bias in the endowment change, $\Delta_{AB}\hat{L}$, did in the two-sector economy. The 3-sector economy equivalent of (18) is thus:

$$\Delta_{AB}\hat{w} = \beta_S\left(\frac{-\beta_R}{\varepsilon + \eta}\{\Delta_{AB}\hat{L}^T + \Delta\mathbf{J}\} + \Delta_{12}\hat{\chi}\right).$$

Substitution, using (25) and related definitions, yields the Section 6 DiD equations after simplification.