

Building the City: From Slums to a Modern Metropolis

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We model the building of a city, estimate parameters of the model, and calculate welfare losses from institutional frictions encountered in changing land-use. We distinguish formal and slum construction technologies; in contrast to slums, formal structures can be built tall, are durable, and non-malleable. As the city grows areas are initially developed informally, then formally, and then redeveloped periodically. Slums are modelled as a technology choice; however, institutional frictions in land markets may hinder their conversion to formal usage that requires secure property rights. Using unique data on Nairobi for 2003 and 2015, we develop a novel set of facts that support assumptions of the model, estimate all parameters of the model, and calculate welfare losses of conversion frictions. We track the dynamic evolution of the city and compare it with model predictions. In the core city formal sector, about a third of buildings were torn down over 12 years and replaced by buildings on average three times higher. For slums in older areas near the centre, even after buying out slumlords, overcoming institutional frictions would yield gains amounting to about \$18,000 per slum household, thirty times typical annual slum rent payments.

Key words: City, Urban growth, Slums, Urban structure, Urban form, Housing investment, Capital durability.

JEL Codes: O14, O18, R1, R3, H

1. INTRODUCTION

The urban population of Sub-Saharan Africa is predicted to increase from 450 million to 1,250 million over the next 30 years (UN, 2018). The populations of many large cities are increasing by about 50% every 10 years, implying a huge demand for increased building volumes. World Bank (2006) suggests that about two-thirds of a country's non-governmental capital stock are buildings, and urban construction and maintenance are a large and rising share of many countries' total investment. Yet, we know little about this investment process that reshapes cities and redeploys nations' capital stocks. African cities are subject to rapid redevelopment and expansion in the

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formal building sector; while at the same time, there is formalization of some slums, densification of others, and spread of new slums at city edges. In parts of cities, formal sector high-rise buildings on high price land are adjacent to low-level slums with limited returns to land, suggesting inefficient land-use and distorted investment decisions.

This paper models the process of a city's growth and applies the model to a unique data set we have developed on Nairobi, the capital of Kenya. The paper does four novel things. First, we develop a model of the built environment of a growing city. Given the difficulty of having both continuous time and space in the same model, especially with durable capital, virtually all models of cities are static. Fujita (1982) has an early attempt at a dynamic model and Braid (2001) put forth the first complete example of a dynamic monocentric city model with durable capital. To study the development of rapidly growing cities in middle- and low-income countries a full dynamic framework is essential. We extend and enrich the base model designed for developed countries by adding a slum sector and a role for geographical and institutional frictions, and we develop clear propositions about city growth.

Dynamics involve spread of a city's land area, intensified land use within the city, and demolition and reconstruction with increased density and building height. Our model captures these features, with a key distinction between formal and informal (slum) sectors. Formal buildings involve sunk capital costs, can be built tall, cannot be modified once constructed, and involve investment decisions based on expected future rents. As the city and housing prices grow formal sector buildings are periodically demolished and redeveloped to a greater height. In contrast, in the informal sector, buildings generally are much more malleable and built with non-weight-bearing materials. They are built low, with high land intensity. The building volume delivered by slums increases through time not by building taller, but by increases in crowding and already high cover-to-area ratios. New slums appear near the expanding city edge as the highest and best use of that relatively cheap edge city land. Within the city, the model predicts that as housing prices rise, there should be on-going conversion of older slums to formal sector use.

Second, we assemble a rich data set on the built fabric of Nairobi and present aspects of the 2015 cross-section and its evolution since 2003. While there is work on the USA using census data of building ages (Brueckner and Rosenthal, 2009), we know of no work which utilizes citywide data on individual buildings, with demolition, redevelopment, and infill to detail the changes in the urban landscape. For the Nairobi data set, we have tracings of all buildings in the city from aerial photo images for 2003 and 2015, which give a precise delineation of the built environment. We develop an algorithm to overlay the two cross-sections of polygons and determine which building footprints are unchanged, which buildings were demolished and/or redeveloped and where and to what extent infill occurs. Building heights are derived from LiDAR data allowing us to measure building volumes. We also have formal sector vacant land prices for 2015, housing rent data for the formal and informal sectors in 2012, and data on land quality throughout the city, covering slope, ruggedness, water cover, and the like.

With these data, we construct a rich picture of the built environment of Nairobi, both in the spatial cross-section and its evolution through time. The UN (2018) reports a population for the agglomeration of just over 4 million, a population that grew at 4.4% p.a. from the 1999 to 2009 census Kenya (1999, 2009). We find that the built volume of the 2015 city area increased at 4.0% p.a., expanding by 60% between 2003 and 2015. Within just the 2003 spatial area of the city, overall volume growth was an astounding 47%, accounting for two-thirds of total volume growth within the 2015 city area. Much of this growth involves formal sector redevelopment. For example, at 3–4 km from the centre, redevelopment increased volume by 30%, achieved by tearing down one third of buildings and reconstructing more intensively. We find that newly

redeveloped buildings are about triple the height of those torn down. Slower growing developed country cities generally have nowhere near this degree of change.¹

Nairobi's slums deliver similar built volumes per unit land as the formal sector, and house just under 30% of the population in both 1999 and 2009, similar to [UN-Habitat's \(2016\)](#) estimates for many developing country cities. Slum volume growth of 50% between 2003 and 2015 was slower than formal sector volume growth of 61%. As noted above, new slums appear on the city edge, often on land with private property rights, highlighting the on-going role for slums to provide low cost housing in developing countries. However, apart from growth at the edge, a key feature is the persistence of several slum areas on high value land near the city centre, most notably Kibera, often referred to as Africa's largest slum.

Third, we estimate all the parameters of the dynamic model based on the characteristics of the 2015 cross-section of the built environment and on land sale and house-rent data, together with the discount rate that we take from World Bank data on Kenya. These parameters describe the intensity of land-use in the informal and formal sectors and gradients telling us how prices, building volumes and heights change with distance from the city centre in both sectors. We then use these parameters and the model to make predictions about the evolution of the built environment and population from 2003 to 2015 and compare these with the evolution we see in the data.

Finally, we calculate the welfare cost of inefficient land-use from delayed formalization of slums near the city centre. This highlights the role of policy for fast growing cities with major land market failures that deter investment. We argue that institutional frictions in Nairobi delay timely redevelopment of inner-city slums and quantify the losses from such delays. For older slums at 3–5 km from the centre, we estimate that the city could pay off illegal slumlords operating in these slums for the value of their land (reflecting their profits) in perpetual slum use. Even after that, formalization would still bring a gain of about \$18,000 per slum household, in a context where slum households spend on average about \$500–700 p.a. on housing. Our calculations do not capture the wider social costs and benefits of formalization which we discuss, but they do quantify the direct land-use efficiency loss of delayed formalization.

1.1. *Context and literature*

Our modelling judgements are based on several key features of Nairobi, features that we think are applicable to many other developing cities. First, we treat Nairobi as a monocentric city. Monocentricity is the workhorse assumption of urban economics, including contemporary papers such as [Combes et al. \(2019\)](#) on modern French cities. As we will see, Nairobi appears to be primarily monocentric with building height, volume, and land prices all declining sharply with distance from the centre. This monocentricity appears to be a feature of many major African cities. [Lall et al. \(2017, p 21–23\)](#) show that African cities like Nairobi, Addis Ababa, Dar es Salaam, and Kigali are much more monocentric than Paris, London, Barcelona, or Atlanta. For our own sample of world cities similar to Nairobi in size, in [Supplementary Appendix Section SA2.4](#), we show that developing country cities and Nairobi in particular display greater monocentricity than most developed country cities, with density dropping off three times more rapidly from the centre.

1. To give perspective on the demolition rate of one-third, in the USA, the American Housing Survey data give demolition and removal by disaster (fire, hurricane, and the like). Annual rates of demolition and removal by disaster range from 0.5% to about 1.2%. For 12 years, this would involve 6–15% of building removal. Nairobi is thus 2–5 times that.

The second feature is that there are relatively well-functioning markets for the allocation of housing within both the formal and informal sectors, but a problem in the conversion of informal to formal sector land-use. While the two sectors offer very different qualities of housing, both are overwhelmingly rental markets; 87% of Nairobi residents overall and 89% of slum dwellers rent, proportions similar to those in many African cities.² The rental markets allocate land between occupants, but the individuals who control slum land and to whom rents are paid often have insecure land rights. This does not impede the functioning of rental markets, but insecure rights do deter those who control the land from sinking the capital required to construct formal sector buildings. Without security of ownership such investments are too risky to be undertaken. This is a problem particularly on government owned—but not managed—lands nearer the city centre in Nairobi. Politically well-connected slumlords ‘illegally’ operate vibrant rental markets, but they cannot switch these lands to formal sector use because that would open their control of the land to challenge. Their ability to operate the slum rental market involves corruption, and a complex political history, as we describe in Section 6. Marx et al. (2013) highlight the importance of “investment inertia”, where small groups of powerful individuals control large slum housing markets. Formalization and similar initiatives would challenge entrenched land control, so the status quo of informal status persists. These situations seem to be common in older slum lands near city centres in many developing country cities in Asia and Africa.³ These cities exhibit the oft-photographed hotchpotch of land uses, with tall formal sector buildings bordering pockets of single storey corrugated iron sheet slum housing in the urban core as institutional frictions prevent conversion of these slums to formal sector development. It is this that motivates our modelling of distinct formal and slum sectors in this context, following the literature that generally makes the same sharp distinction, even if the line between informal and formal sector housing may be fuzzy at times.

Similar institutional barriers to formalization arise in slum areas around the edge of Nairobi. Here a higher proportion of land is privately owned. However, some owners may have only possessory rights which need to be converted to formal modern title prior to sinking capital in formal sector development. This again involves formalization costs, in this case in the form of fees, legal costs, delay, and possibly bribes. Frictions arise for natural as well as institutional reasons, including poor geographic conditions such as steep slope, ruggedness, marshes, and exposure to flooding. We examine these empirically to establish when and where each set of frictions matters. Our results suggest that institutional rather than natural frictions play the dominant role for older slums near the centre.

On other literature, we note that recent papers have varying perspectives in modelling the informal-formal sector distinction. For the Brazilian context, in Cavalcanti et al. (2019) slum owners make a choice between formalizing, which involves paying for secure rights and incurring taxes and regulation, versus staying in the unregulated informal sector with insecure land rights. We do not focus on taxes and regulation, because in Nairobi tax systems generally apply to both

2. From UN-Habitat (2011) on Ghana about 75% of residents in Accra and Kumasi rent. From authors’ own calculations from IPUMS about 40% in Dar es Salaam rent. At a country level, among urban dwellers as defined in IPUMS for Ethiopia (2007), Nigeria (2010), and Uganda (2002), respectively, 51%, 60%, and 58% rent, where we expect higher amounts in primate cities. So while the urban Kenya rate in IPUMS is 56%, in Nairobi as noted in the text it is 87%. Some other counties including Mozambique and South Sudan have higher ownership rates.

3. There is no central source of information on slum locations and sizes. We explored sources including the UN-Habitat Slum Almanac (2016), Davis (2006), and the repository of CIESEN at Columbia University. We compiled a long list of slums near city centres. Well-known examples include Dhavari in Mumbai with population estimates of 7–800,000; Manshiyat Nasser (“Garbage City”) in Cairo with its edge within 3 km of the centre and population up to 800,000; and Ga Mashie and Old Fadama in Accra housing about 400,000. City centre slums in Jakarta and Dar es Salaam city slums are analysed in Harari and Wong (2019) and Michaels et al. (2019).

sectors and there is very low compliance in either sector. Similarly, land use regulations appear not to bind.⁴ Two recent papers investigate whether slum upgrading programmes inhibit timely formalization. For Jakarta, Harari and Wong (2019) argue that slum improvement programmes serve to delay redevelopment of slums. Similarly, Michaels et al. (2019) argue that, compared to untreated neighbouring slums, improved slums either look no different than untreated ones 2–3 decades later, or have poorer characteristics, in Dar es Salaam. Finally, Getcher and Tsivanidis (2018) argue that in Mumbai very large formal sector developments can spur redevelopment of nearby slum areas. Our work complements these papers looking at the dynamic aspects involved in investing in land and using the dual structure of urban land markets that we think is relevant to many developing country situations.⁵

The basic model and core theoretical results are set out in Section 2 of the paper. Section 3 presents data and estimates key empirical relationships. Section 4 backs out the parameters of the model, based on empirical evidence, and validates key assumptions of the model. Section 5 analyses the evolution of the built environment and compares model predictions with the data. Section 6 develops the welfare analysis to derive costs of misallocation of land to older slums, and Section 7 concludes.

2. THEORY

In this section, we develop the model of a growing city, focusing on investment decisions and consequent patterns of land-use and built structures. Sections 2.1 and 2.2 describe housing demand and supply, introducing preferences and technologies for informal (slum) and formal sector buildings. We derive the investment decisions and consequent land-rent that would be earned if land at a particular place and date were in informal or formal sector use (or left for agricultural use outside the city). In Section 2.3, we turn to landholders' dynamic optimization problems of choice of land-use, i.e. whether, at each place and date, land should be occupied by slums or formal sector buildings, and the timing of formal sector redevelopment. We look first at a particular point in the city and examine its evolution through time as it transitions from agricultural use to informal development, then formalizes and goes through successive waves of formal sector demolition and reconstruction. Section 2.4 draws out the way in which his path varies across points in the city, giving a description of both the cross-section of the city and its evolution through time. Section 2.5 adds frictions to the transition process, in particular spatial heterogeneity in formalization and construction costs. Together, these analytical sections provide the relationships upon which empirics, estimation of key parameters and welfare analysis are based.

4. In Nairobi, land-use regulations seem to be largely ignored (Mwaura, 2006) and property taxation of formal sector lands has very limited implementation and compliance (Kelly, 2003). Narubi (2015) shows for property taxes compliance in Nairobi in 2014/15 was under 7%, and the collection ratio was about 16%, suggesting only some bigger commercial properties pay these taxes. Furthermore, from the 2010 cadastre for Nairobi county, most slum lands are assessed these same taxes. On regulations, the minimum lot size is generally 500 m², and never less. In Supplementary Figure SA2.2 of the Appendix, from a full cadastre of the formal sector, we show that modal lot size is at 180 m², 54% of all buildings are in violation of the regulation, and that there is a long right tail for commercial buildings. Most particularly, the curve for that distribution has no mass, step, blip, or kink around 500 m².

5. Additionally, Selod and Tobin (2018) model owners as choosing between different gradations of property rights at different costs, an issue relevant for some contexts. Ferreira et al. (2016) on Brazil focus on modelling how slums may be stepping-stones in skill formation and growth of cities.

2.1. Bid-rents and housing demand

The value of a unit of housing varies across the city and through time, and we start by deriving the bid-rent for housing at places x and date t . A representative household has preferences between consumption of housing and other goods (the numeraire) described by $U(s(x,t)a, w(x,t) - q(x,t)s(x,t))$. In this expression, $s(x,t)$ is the quantity of housing space consumed by an individual living at place x at date t , $q(x,t)$ is its observed rental price, and a is an amenity parameter which will be discussed further below. Household income available for housing and purchase of other goods is $w(x,t)$. We take the function $w(x,t)$ to be exogenous and assume that it is increasing in t and weakly decreasing in x . The former is based on the city using labour to produce output with constant returns to scale and experiencing productivity growth relative to the rest of the economy.⁶ The latter can be a simple labelling of places. However, we generally interpret x as distance from the city centre, in which case $w(x,t)$ is interpreted as wages at the centre minus commuting costs to distance x .⁷ We impose a particular functional form on $w(x,t)$ from Section 2.4 onwards.

The city is “open”, meaning there is free migration between the city and outside, where utility takes exogenous value u_0 . Equilibrium rent leaves all households indifferent between living at each occupied place x in the city or receiving outside utility u_0 . Housing rent $q(x,t)$ therefore satisfies $U^*(q(x,t)/a, w(x,t)) = u_0$, where $U^*(\cdot)$ is the indirect utility function (each household’s location and $s(x,t)$ optimally chosen). Since the amenity factor enters the utility function multiplicatively with $s(x,t)$, it enters indirect utility in the form $q(x,t)/a$. Henceforth, we work with rent per unit amenity, defined as $p(x,t) = q(x,t)/a$, and referred to as space-rent. The net wage $w(x,t)$ therefore determines space-rent through the relationship

$$U^*(p(x,t), w(x,t)) = u_0, \text{ for example, } p(x,t) = (w(t,x)/u_0)^{1/\beta}, \quad (1)$$

where the example has Cobb–Douglas preferences with housing share β and hence chosen quantity $s(x,t) = \beta w(x,t)/ap(x,t)$ (see [Supplementary Appendix Section SA1.1](#)). Outside utility is exogenous and will henceforth be suppressed in notation.⁸ We assume that housing-space is a normal good, implying that space-rent $p(x,t)$ is increasing in $w(t,x)$, and hence increasing with t and weakly decreasing with x . The dynamics of the model are therefore driven by urban productivity growth that increases $w(t,x)$, i.e. shifts this horizontal labour demand curve up through time. This increases space-rent, and the consequent growth of city population is determined by the supply response of urban housing.

We note that, while we have different types of building technology, we do not model different types of households. Were we to do so, land-use would be determined by the upper envelope of the land-rent functions generated by different households.⁹ In our empirical work, land-rents in some parts of the city are systematically lower for land in informal use than for equivalent land in formal use. This is indicative of inefficient land-use, regardless of whether the land-rent function is that of a single household type or the upper envelope of many types. We will return to discussing the relevance of having different types of households in Section 6 when we conduct our welfare analysis.

6. Agglomeration economies are not included in this model. The price of output is exogenous to the city, and any variations in the output price have the same effect as changes in physical productivity.

7. See [Supplementary Appendix Section SA1.1](#) for derivation of these relationships. This is as in the standard urban model, where all households commute to the central business district (CBD). Newer approaches offer a richer model of commuting and the dependence of net wages on location (e.g. [Ahlfeldt et al. 2015](#)). We assume also that households have no other source of income, implying that land-rents are received by non-resident landowners or slumlords.

8. u_0 is potentially time varying, but in later analysis it is only the exogenous ratio $w(x,t)/u_0$ that matters.

9. See for example [Duranton and Puga \(2015\)](#).

2.2. Building technology and housing supply

There are two distinct building technologies, formal and informal, which supply building volume per unit land in different ways. The formal sector (F) can build tall, and the informal sector (I) can “crowd”, increasing cover, i.e. the proportion of land covered by building footprints. The volume of building that a technology can supply on a unit of land at a particular place, x , at time t , is the product of height and cover, $v_i(x, t) = h_i(x, t)c_i(x, t)$, $i = I, F$. Formal sector buildings are durable structures, which incur sunk costs of construction and increasing marginal cost of building tall. The informal sector is unable to build tall because the materials used are not sufficiently load-bearing, this also giving a “malleability”, enabling structures to be rapidly modified and implying that construction costs are not sunk. Informal floor-space can be added at constant marginal cost, but crowding reduces amenity and hence housing rent.

This distinction captures the difference in sunk costs associated with each housing type, and its technological basis accords well with the difference between slums and formal sector areas in Nairobi. The 2009 Nairobi Census indicates that, in slums, the majority (about 52%) of households live in buildings where walls are corrugated iron sheets which can be easily reconfigured like Meccano parts; much other slum housing involves mud construction (about 19%) and other material with short duration (Kenya, 2009). These materials are not sufficiently load-bearing to allow much height, and our data show that 85% of slum buildings are under 5 m tall. In contrast, 84% of formal sector households live in a building that is made of stone or some type of brick or block, many with substantial height. The main missing category in both sectors is wood construction, where wood buildings in slums are mostly very different quality structures than those in the formal sector. While, in reality, the distinction between slums and formal sector developments can be ambiguous, the binary distinction is analytically tractable. We also think the sharp distinction captures the key features of Nairobi and is supported by the literature, including Taubenbock et al. (2018) who analyse the morphology of slums for a large selection of cities worldwide.

2.2.1. Informal sector. Informal sector construction materials are malleable and construction costs are a flow, occurring continuously through the life of the structure. This can be thought of as either the rental on ‘Meccano parts’ used in construction or as the cost of material whose life is one instant. This sector is unable to build tall, so height is fixed at $h_I = 1$. It can, however, change at any instant the proportion of each unit of land that is covered with buildings, $v_I(x, t) = c_I(x, t)$. Construction costs per unit volume in this sector are constant κ_I , so construction costs per unit land are $\kappa_I v_I(x, t)$. However, crowding more building onto land in slums reduces housing amenities, which we capture by assuming that the amenity is a diminishing and convex function of cover per unit land, so $a(v_I(x, t))$, $a' < 0$, $a'' > 0$. If land at x, t is in informal use, then land-rent is space-rent times amenity minus construction cost, all times volume per unit land,

$$r_I(x, t) = [p(x, t)a(v_I(x, t)) - \kappa_I]v_I(x, t). \quad (2)$$

The volume of housing supplied at any instant is chosen to maximize land-rent, taking space-rent $p(x, t)$ as exogenous and internalizing the effect of crowding on amenity.¹⁰ The first-order condition equates marginal revenue to marginal cost,

$$\partial r_I(x, t) / \partial v_I(x, t) = p(x, t)a(v_I(x, t)) [1 + v_I(x, t)a'(v_I(x, t)) / a(v_I(x, t))] - \kappa_I = 0. \quad (3)$$

10. A simplification is that we do not explicitly model this as an externality in which each developer’s choice of crowding affects neighbours.

Generally, we assume that informal amenity is iso-elastic in cover, $a(v_I(x,t)) = a_I v_I(x,t)^{(1-\alpha)/\alpha}$, $\alpha > 1$. Then the chosen volume of housing and maximized value of land-rent are

$$v_I(x,t) = [a_I p(x,t) / \kappa_I \alpha]^{\alpha/(\alpha-1)}, \quad (4)$$

$$r_I(x,t) = (\alpha - 1) \kappa_I v_I(x,t) = \kappa_I (\alpha - 1) [a_I p(x,t) / \kappa_I \alpha]^{\alpha/(\alpha-1)}. \quad (5)$$

The loss of amenity due to crowding, α , is a key parameter. It will be recovered in the empirics based on how informal sector volume in equation (4) varies with distance from the city centre. The parameter also determines the division of revenue between land-rent and construction costs. Equation (5) indicates that land-rent is multiple $(\alpha - 1)$ of construction costs, so total space-rent earned per unit land is divided with proportion $(1 - 1/\alpha)$ going as land rent, and $1/\alpha$ to construction. Iso-elasticity implies further that informal sector space-rent adjusted for amenity is constant throughout the city at value

$$p(x,t) a_I v_I(x,t)^{(1-\alpha)/\alpha} = \kappa_I \alpha. \quad (6)$$

Essentially, and as will be confirmed later in the space-rent data, increased crowding near the city centre offsets the advantage of improved access to the centre.

2.2.2. Formal sector. The formal sector differs in that buildings are durable, construction is a sunk cost, and volume is achieved by varying height, not cover. Height is chosen at date of construction, denoted τ_i , so $h_F(x, \tau_i)$ is fixed for the life of the structure, i.e. until demolition at date τ_{i+1} , where subscript $i = 1, 2, \dots$ denotes successive redevelopments (or ‘generations’) of formal structures. For simplicity, but also based on the cross-section data, we assume that formal sector land cover is uniform at $c_F = 1$, so the volume of building at x developed (or redeveloped) at τ_i is constant at $v_F(x,t) = v_F(x, \tau_i) = h_F(x, \tau_i)$ for $t \in [\tau_i, \tau_{i+1}]$. Construction costs per unit land are one-off and sunk, and are an increasing and convex function of height (= building volume) on that land, $k(v_F(x, \tau_i))$, $k' > 0$, $k'' > 0$. Demolition incurs no costs.

We assume there is no amenity loss or gain from building tall, so we set $a_F = 1$. The space-rent of a unit of building volume is $p(x,t)$, exogenous to the developer. The present value of land-rent that accrues over the life of a structure, $t \in [\tau_i, \tau_{i+1}]$, discounted to construction date τ_i at interest rate ρ is denoted $R_F(x, \tau_i)$. With sunk costs $k(v_F(x, \tau_i))$ and volume fixed at the date of construction this is

$$R_F(x, \tau_i) = v_F(x, \tau_i) \int_{\tau_i}^{\tau_{i+1}} p(x,t) e^{-\rho(t-\tau_i)} dt - k(v_F(x, \tau_i)). \quad (7)$$

We define the ratio of the present value of space-rent per unit volume over its life relative to space-rent at date of construction as

$$\Phi(x, i) \equiv \int_{\tau_i}^{\tau_{i+1}} [p(x,t) / p(x, \tau_i)] e^{-\rho(t-\tau_i)} dt, \quad (8)$$

so $R_F(x, \tau_i) = p(x, \tau_i) \Phi(x, i) v_F(x, \tau_i) - k(v_F(x, \tau_i))$. The integral $\Phi(x, i)$ is akin to the ‘value-to-rent ratio’ on a newly constructed property in the terminology of the real-estate literature (noting the time horizon in (8) is cut at τ_{i+1}).

If land at place x undergoes formal sector development (or re-development) at date τ_i , the volume chosen is intuitive and given by

$$\partial R_F(x, \tau_i) / \partial v_F(x, \tau_i) = p(x, \tau_i) \Phi(x, i) - k'(v_F(x, \tau_i)) = 0. \tag{9}$$

If the cost function is iso-elastic, $k(v_F) = \kappa_F v_F^\gamma, \gamma > 1$, then chosen volume and the maximized present value of land-rent are ¹¹

$$v_F(x, \tau_i) = \left[\frac{p(x, \tau_i) \Phi(x, i)}{\kappa_F \gamma} \right]^{1/(\gamma-1)}, \tag{10}$$

$$R_F(x, \tau_i) = \kappa_F (\gamma - 1) \left[\frac{p(x, \tau_i) \Phi(x, i)}{\kappa_F \gamma} \right]^{\gamma/(\gamma-1)}. \tag{11}$$

The diseconomy in building taller, γ , is another key parameter and will be estimated based on how the height of newly constructed buildings and hence volume in equation (10) varies with location as space-rent varies.

To define land-rent share, it is useful to have a continuous flow measure of land-rent, given by amortizing the one-off construction cost continuously over the life of the structure. If amortization is a constant proportion μ of revenue over the life of the building, i.e. $\mu p(x, t) v_F(x, \tau_i)$, then construction costs are covered by setting μ to satisfy $\mu p(x, \tau_i) \Phi(x, i) v_F(x, \tau_i) = k(v_F(x, \tau_i))$. With iso-elastic cost function the amortization rate is then $\mu = 1/\gamma$.¹² Flow land-rent net of amortization is therefore fraction $(1 - 1/\gamma)$ of gross revenue earned by land and structure together. This land-rent, net of amortization, is place, time, and date of development specific, and we denote it $r_F(x, t, \tau_i)$; it is given by

$$r_F(x, t, \tau_i) = (1 - 1/\gamma) p(x, t) v_F(x, \tau_i) = (1 - 1/\gamma) p(x, t) \left[\frac{p(x, \tau_i) \Phi(x, i)}{\kappa_F \gamma} \right]^{1/(\gamma-1)}. \tag{12}$$

Land-rents are therefore fraction $(1 - 1/\gamma)$ of space-rent in the formal sector, and fraction $(1 - 1/\alpha)$ in the informal sector (equation 5), relationships that we will use later in the paper. Notice also that, comparing (11) and (12) at date $t = \tau_i$,

$$r_F(x, \tau_i, \tau_i) = R_F(x, \tau_i) / \Phi(x, i). \tag{12a}$$

2.3. Land development and construction phases

Equations (5) and (11) give the maximized returns to respectively informal or formal sector use of land at place x and date t . We now look at the choice of where and when land is in one or other of these uses. We pose this as the decisions of a landholder of when to develop informal structures and when to develop or redevelop formal structures on land at place x . At some date (say time 0) the present value of land-rent at location x that has not yet been developed is

$$PV(x) = \int_0^{\tau_0} r_0 e^{-\rho t} dt + \int_{\tau_0}^{\tau_1} r_I(x, t) e^{-\rho t} dt + [R_F(x, \tau_1) - D(x)] e^{-\rho \tau_1} + \sum_{i=2}^{\infty} R_F(x, \tau_i) e^{-\rho \tau_i}. \tag{13}$$

11. The iso-elastic form implies an elasticity of substitution between land and capital (i.e. construction cost) of unity. This is at the centre of the range suggested in by Ahlfeldt and McMillen (2014).

12. Using $k(v_F) = \kappa_F v_F^\gamma$ the condition $\mu p v_F \Phi = k(v_F)$ becomes $\mu p \Phi = \kappa_F v_F^{\gamma-1}$, and using equation (11) to substitute for v_F gives $\mu = 1/\gamma$.

The first term is the present value of rent earned while the land is undeveloped land (flow rent r_0 which we take to be constant), discounted at rate ρ and accruing up to the date of first development, denoted τ_0 . The second term is the present value of rent while in informal urban use during interval $[\tau_0, \tau_1]$. The first formal sector development, occurring at date τ_1 , yields rent and incurs a potential one-time fixed cost $D(x)$ capturing frictions to formal development, such as unclear land rights or costs of land preparation; we discuss these further in Section 2.5. The final term in equation (13) gives the discounted value of land-rents earned over the lives of consecutive formal sector buildings, constructed at dates $\tau_2, \tau_3 \dots$. Equation (13) assumes this sequence of development, and we will establish conditions under which this sequence is followed. Our key results emerge from solving this dynamic optimization problem and we proceed in stages, so as to explain results.

Dates of development and redevelopment are chosen to maximize $PV(x)$. For the first development the optimal τ_0 simply equates flow land-rents on undeveloped and informal land,

$$\frac{\partial PV(x)}{\partial \tau_0} = e^{-\rho t} [r_0 - r_I(x, \tau_0)] = 0, \quad (14)$$

which, with iso-elasticity and using equation (5) is,

$$r_0 = \kappa_I(\alpha - 1) [a_I p(x, \tau_0) / \kappa_I \alpha]^{\alpha/(\alpha-1)}. \quad (14a)$$

Since space-rent $p(x, t)$ is increasing through time, for each place x there is a unique date τ_0 at which informal development commences.

The first formal development takes place at date τ_1 satisfying (see [Supplementary Appendix Section SA1.2](#))

$$\frac{\partial PV(x)}{\partial \tau_1} = e^{-\rho \tau_1} [r_I(x, \tau_1) - p(x, \tau_1) v_F(x, \tau_1) + \rho \{k(v_F(x, \tau_1)) + D(x)\}] = 0. \quad (15)$$

Using the expression for land-rent (12) and formal volume (10), together with iso-elastic construction technology this can be written as

$$r_F(x, \tau_1, \tau_1)(\gamma - \rho \Phi(x, i)) / (\gamma - 1) - \rho D(x) = r_I(x, \tau_1). \quad (15a)$$

This gives a unique switching point from informal to formal if $\alpha > \gamma$ (from equations 5 and 12), which we assume and which is a condition that our estimates of α and γ clearly satisfy.

The first redevelopment of formal land is at date τ_2 satisfying (see [Supplementary Appendix Section SA1.2](#))

$$\frac{\partial PV(x)}{\partial \tau_2} = e^{-\rho \tau_2} [p(x, \tau_2) v_F(x, \tau_1) - p(x, \tau_2) v_F(x, \tau_2) + \rho k(v_F(x, \tau_2))] = 0.$$

Generalising this for all redevelopments gives:

$$p(x, \tau_{i+1}) [v_F(x, \tau_{i+1}) - v_F(x, \tau_i)] = \rho k(v_F(x, \tau_{i+1})), \quad \text{for } i \geq 1. \quad (16)$$

Again using equations (12), (10), and iso-elastic construction technology this can be written as

$$r_F(x, \tau_{i+1}, \tau_{i+1})(\gamma - \rho \Phi(x, i+1)) / \gamma = r_F(x, \tau_{i+1}, \tau_i). \quad (16a)$$

Intuition on these redevelopment dates can be seen from inspection of equation (16).¹³ This says that demolition and reconstruction occur at the date at which the instantaneous revenue gain from the change in volume equals the interest cost of the construction expenditure incurred. Equations (14)–(16) implicitly define the dates at which sites are (re-)developed. These equations, together with the definition of the value-to-rent ratio, $\Phi(x, i)$ in equation (6), form the basis of the analysis of the next sub-section.

2.4. *Analysis*

What do we learn from the characterization of development stages given above? Assuming space-rents grow at constant exponential rate $\hat{p} > 0$ yields analytical results, where this growth is driven by the growth in urban wages relative to the outside option, derived from equation (1) (see [Supplementary Appendix Section SA1.1](#)). We look first at the time series development of a particular place x , and then at the urban cross-section.

2.4.1. Urban dynamics at any location. The process of redevelopment of land that has been formalized is summarized as follows:

Proposition 1 *If formal sector construction costs are iso-elastic in height (with elasticity γ), space-rents are growing at constant exponential rate $\hat{p}, \rho(\gamma - 1)/\gamma > \hat{p} > 0$, and agents have perfect foresight then:*

- (i) *The value-to-rent ratio takes constant value Φ and the time interval between successive formal redevelopments is constant $\Delta\tau$,*

$$\Phi = \int_0^{\Delta\tau} e^{(\hat{p}-\rho)t} dt = \frac{1 - e^{(\hat{p}-\rho)\Delta\tau}}{\rho - \hat{p}}, \quad \Delta\tau = \frac{\gamma - 1}{\hat{p}} \ln \left[\frac{\gamma}{\gamma - \rho\Phi} \right]. \tag{17}$$

- (ii) *Successive rounds of formal sector building have greater volume (height) by a constant proportional factor*

$$\frac{v_F(x, \tau_{i+1})}{v_F(x, \tau_i)} = e^{\hat{p}\Delta\tau/(\gamma-1)} = \frac{\gamma}{\gamma - \rho\Phi} > 1. \tag{18}$$

- (iii) *Land value (capital value of a unit of land) in the formal sector at location x newly redeveloped time τ_i is given by*

$$PV_F(x, \tau_i) = r_F(x, \tau_i, \tau_i) \Phi / \left[1 - e^{-\left(\rho - \frac{\hat{p}\gamma}{\gamma-1}\right)\Delta\tau} \right], \tag{19}$$

equivalent to constant growth rate per unit time of $\hat{p}\gamma/(\gamma - 1)$.

The first part of this proposition comes from integrating equation (8), using it in equation (16a), and noting that there is a unique solution solving the two parts of equation (17) with Φ and $\Delta\tau$

13. The switch dates in equations (15) and (16) are *not* when flow land-rents are equalized. This is because land-rents (net of amortization) jump as volume increases at date of construction. The terms in γ and $\rho\Phi$ make the appropriate adjustments. Notice that in equation (15) the term $(\gamma - \rho\Phi(x, i))/(\gamma - 1) = 1$ in a stationary model with $\hat{p} = 0$ and the first-generation development lasting in perpetuity.

constant over time and space. The second part follows from equation (16a), noting that, from equation (12) the ratio of land-rents at date τ_{i+1} on vintages $i+1$ and i is simply the ratio of volumes. Part (iii) gives the capital value of a unit of land in the formal sector at location x at newly redeveloped time τ_i derived by using $R_F(x, \tau_i) = r_F(x, \tau_i) \Phi$ in equation (12), and summing over infinitely repeated cycles of redevelopment; the sum is finite if $\rho > \hat{p}\gamma/(\gamma - 1)$. Flow land-rent $r_F(x, \tau_i, \tau_i)$ increases at rate \hat{p} during the life of a building, and jumps up by factor $e^{\hat{p}\Delta\tau/(\gamma-1)} = \gamma/(\gamma - \rho\Phi) > 1$ at date of redevelopment, due to the greater building volume. The average growth of the land-value is greater than that of space-rent in anticipation of these jumps in volume. Relationship (19) between current land-rent and land-value (reflecting future rent increases) will be used in Section 3 to back out an estimate of \hat{p} , the rate of increase in housing rents. Note also that the proposition tells us that, if the rate of growth of space-rent is the same in all formalized locations, then so too are Φ , $\Delta\tau$, and the growth rates of building volume and land-value.

Proposition 1 deals with redevelopment of land that has previously been formalized. The timing of informal and first formal development are given by equations (14) and (15), and hinge on comparisons between r_0 , $r_I(x, t)$, and $r_F(x, t, t)$. With Φ now constant from proposition 1 we have:

Proposition 2 *If formal sector construction costs are iso-elastic in height (with elasticity γ), informal amenity is iso-elastic in volume (with elasticity α), space-rents are growing at constant exponential rate \hat{p} , $\rho(\gamma - 1)/\gamma > \hat{p} > 0$, agents have perfect foresight and $D(x) \geq 0$ then:*

- (i) *Informal development exists for an interval of time $[\tau_0, \tau_1]$, positive if and only if τ^* solving $r_0 = r_F(x, \tau^*, \tau^*)(\gamma - \rho\Phi)/(\gamma - 1) - \rho D(x)$ is greater than τ_0 .*
- (ii) *If informal development occurs, then it precedes formal development.*

The equation in the first part of the proposition implicitly defines τ^* as the date at which it would be profitable to switch undeveloped rural land directly to formal sector development. If $\tau^* > \tau_0$, then the switch date from rural to informal (equation 14) occurs prior to this date, so there exists a period of informal land-use prior to formalization. [Supplementary Appendix Section SA1.3](#) derives the inequality under which this holds, $((r_0 + \rho D)/(\gamma - \rho\Phi))^{(\gamma-1)/\gamma} (\kappa_F/\Phi)^{1/\gamma} \gamma > (r_0/(\alpha - 1))^{(\alpha-1)/\alpha} \kappa_I^{1/\alpha} \alpha/a_I$. As expected, an interval of informality is more likely the lower is κ_I , higher is a_I , the higher is κ_F/Φ , and the lower is r_0 .¹⁴ The second part of Proposition 2 is proved by noting that r_I is increasing at rate $\hat{p}\alpha/(\alpha - 1)$ (equation 6) and r_F is increasing at rate $\hat{p}\gamma/(\gamma - 1)$ (equation 12). Given $\alpha > \gamma$, there is therefore a unique crossing, switching from informal to formal. The assumption $\alpha > \gamma$ means that there are sharper diseconomies to informal sector crowding than to formal building height, and that the share of land-rent in revenue is higher in informal development than in formal.

Figure 1 illustrates these results in a stylized benchmark city without frictions, using model parameters estimated in Section 3.¹⁵ Building volume is given on the vertical axis (log units), and on the horizontal plane axes are time t and location x . Location is distance from the centre, and we discuss the cross-section (variation across x at a given t) in the next sub-section. For the moment, look just at the development of a particular location through time, i.e. fix x and look along a line sloping up and to the right parallel to the t axis. Initially (at low t) this land is rural. Building volume becomes positive at date τ_0 (specific to location x) when informal development takes

14. r_0 determines the date of edge slum development, but has no bearing on subsequent time intervals between this and formalization or subsequent redevelopments.

15. See [Supplementary Appendix Section SA1.4](#) for listing of parameters.

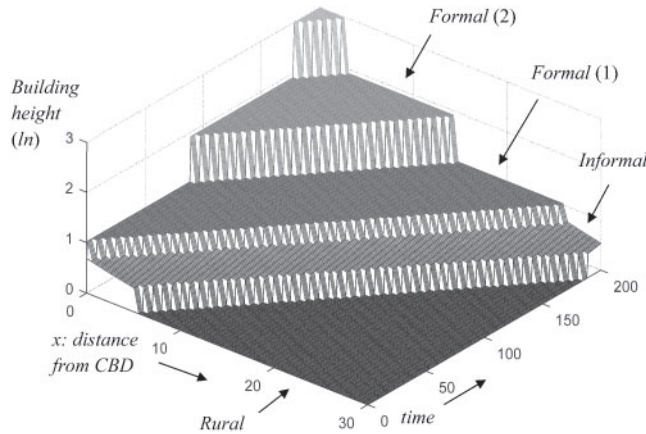


FIGURE 1
Urban development with perfect foresight

place. The volume of informal development increases steadily, as increasing p causes Meccano pieces to be rearranged and building cover to increase. Formal development takes place at τ_1 and, as illustrated, leads to a small increase in volume, indicated by the second step. Subsequent redevelopments occur at fixed time intervals $\Delta\tau$ and bring the same proportionate increase in volume, achieved by building taller. The timing and volume of each of these formal investments is based on perfect foresight about the growth of prices and the date of subsequent redevelopments.

2.4.2. The urban cross-section and its evolution. We have so far concentrated on a single location, x , and now show how development varies across places in the city. We interpret x as distance from the city centre and assume the existence of commuting costs such that wages net of these costs, $w(x, t)$, decrease with distance from the centre at exponential rate θ . In [Supplementary Appendix Section SA1.1](#), we give the underlying assumptions that yield this exponential form. Exponential decline with distance and exponential growth through time imply that space-rents are

$$p(x, t) = \bar{p} e^{\hat{p}t} e^{-\theta x}. \tag{20}$$

The switch points in equations (14–16) can now be interpreted either as giving the date at which place x develops, or the place that develops at date t . The latter interpretation gives the urban cross-section. Proposition 3 states how different stages of development vary across this stylized city as it grows, and Proposition 4 summarizes how this cross-section evolves through time.

Proposition 3 *If formal sector construction costs and informal sector quality are iso-elastic, space-rents are growing at constant exponential rates \hat{p} , $\rho(\gamma - 1)/\gamma > \hat{p} > 0$ and declining with distance at constant rate $\theta > 0$, and agents have perfect foresight then:*

- (i) *The distance from the city centre to the edge of new informal development increases through time according to $dx_0/dt = \hat{p}/\theta$.*
- (ii) *If $D=0$, the distance from the city centre to the outer edge of formal development increases through time according to $dx_1/dt = \hat{p}/\theta$.*

(iii) *The distance between successive formal sector redevelopments, Δx , is constant,*

$$\Delta x = \frac{\gamma - 1}{\theta} \ln \left[\frac{\gamma}{\gamma - \rho \Phi} \right].$$

See [Supplementary Appendix Section SA1.5](#) for proof.

These results are illustrated in Figure 1, where we now fix a date and move along a line parallel to the x -axis. At the city edge land is informal. Moving towards the centre, at the inner edge of informal land there is the most recent formalization; and, as we move in, further places that have been urban for longer have been through more stages of development, with a jump in housing volume and height at each stage. As the city grows, part (i) of the proposition says new informal sector development is pushing continuously into rural land on the fringe. Similarly, part (ii) says the ring of most recent formal sector development is pushing continuously into the inner edge of the informal sector ring and that the width of the ring of informal area, $x_1 - x_0$ is constant through time. Hence, even in a circular city, the share of urban land area that is informal declines as the city gets larger. Part (iii) says that the width of formal sector rings of development is constant. We also note that as we move from the city edge to the inner boundary of slums land, housing quality declines continuously as more investment is crowded onto each piece of land. Thus, iso-elasticity and exponential growth yield simple patterns in a benchmark city without frictions.

2.5. *Spatial heterogeneity and formalization costs*

Cities do not look like the neat pattern described above. Places have idiosyncratic features, and our focus here is on those that affect the ability to formalize land. These include institutional and geographic costs of moving from informal to formal settlement, all of which are summarized in our modelling by the parameter $D(x)$, the fixed cost of formalization at place x .¹⁶

The implications of this spatial heterogeneity are illustrated in Figure 2 in which $D(x)$ varies with distance from the centre, taking random non-negative values (details in [Supplementary Appendix SA1.4](#)). Instead of the sharp edges of Figure 1, stages of development are now locally diffused because of the variation in $D(x)$. This yields “waves of development” in different grey-scale shades in the figure. The leading edge of the formalization wave is places with low D and, within a particular distance band, this is followed a process of infill, with the fraction of the area covered by formal buildings increasing through time. The city cross-section therefore exhibits areas of informality next to formal structures, some of which may have gone through several phases of redevelopment. In the informal areas, volume and crowding increase. This is the pattern that we see in the data, as will be discussed in Section 5.

We know from Proposition 1 that the time interval between each redevelopment at a particular place is fixed, so it follows that if first formal development is later and taller, then so too are subsequent redevelopments.

Proposition 4 *If formalization costs $D(x)$ vary across places, space-rents are growing at constant exponential rates $\hat{p}, \rho(\gamma - 1)/\gamma > \hat{p} > 0$ and declining with distance at constant rate $\theta > 0$, formal sector construction costs are iso-elastic and agents have perfect foresight then:*

16. While we have $D(x)$ as a conversion cost that occurs with formalization whether converting land from slums, farmland or vacant land, there may be an additional cost of clearing slums per se, including eviction costs even from private land. For properties where such costs may be high due to history and specific rights, landholders may skip the slum stage and go from farmland to formalization directly. This element is not added to the model here to limit complexity but is noted again in Section 5 in interpreting data.

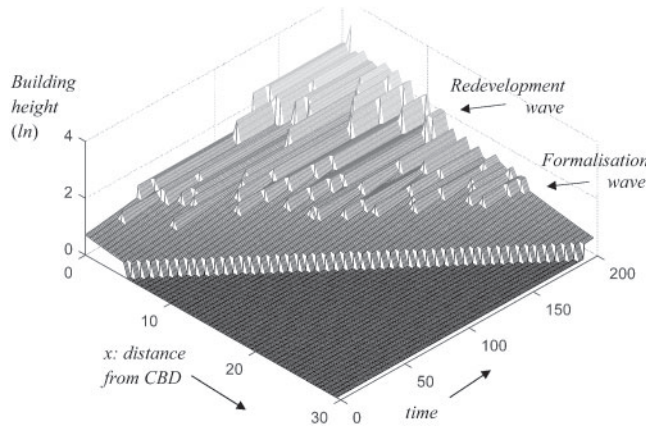


FIGURE 2

The hotchpotch—random variation in fixed costs of development, $D(x)$

- (i) Formal development is delayed for places with high $D(x)$, but proposition 1 continues to hold for each x .
- (ii) Variation in $D(x)$ means that first formal development and subsequent redevelopments may be undertaken in different places at the same date. For such developments, volume (height) and land-value gradients are

$$\frac{dv_F(x, \tau_i)}{dx} \frac{1}{v_F} = \frac{-\theta}{\gamma - 1}, \quad \frac{dR_F(x, \tau_i)}{dx} \frac{1}{R_F} = \frac{-\theta\gamma}{\gamma - 1}. \tag{21}$$

The spatial gradients in the last part of this proposition are derivatives of equations (10) and (11) and will be used in the empirical section of the paper to back-out parameters of the model.

Spatial heterogeneity of formalization costs $D(x)$ supports the observed hotchpotch of adjacent formal and slum buildings. Some of the costs are incurred in securing private property rights (for either leasehold or freehold land) needed to avoid risk of expropriation, to obtain financing and insurance, and to clarify issues such as inheritance, compensation for takings and the like. Other costs arise from the institutional blockages in the long and tortured development of private property rights in Nairobi, as discussed in Section 1. Such frictions involving corruption and a dubious legislative history transferring land to the government have been a critical factor in the city’s development. These frictions vary widely across areas of the city, as we discuss in Section 6.

Spatial heterogeneity in $D(x)$ may be due to geographical as well as institutional factors. Initial construction costs may vary with terrain, for example the costs of draining a swamp or levelling a rugged site for development. We offer empirical evidence for the presence of such effects as discussed in Section 5.1 and [Supplementary Appendix S3](#). We show that poorer geography delays initial development. High price land near that city centre undergoing first development has distinctly poorer terrain compared to the city edge, as all better terrain land near the city centre has already been developed. Evidence also suggests that poor terrain affects initial development costs, rather than costs of redevelopment, since it does not reduce building heights on redeveloped formal sector land.¹⁷

17. Specifically, in [Supplementary Appendix Table SA3.2](#) rebuilding heights are uncorrelated with elevation, presence of water in the neighbourhood, and do not decrease with greater ruggedness.

3. THE BUILT ENVIRONMENT OF NAIROBI

Our empirical work provides evidence on the built fabric of Nairobi and its evolution through time. We use the data and analytical structure to estimate equations characterizing the 2015 cross-section, and these are used in Section 4 to solve for model parameters. In Section 5, we use the model and parameters to make predictions about the dynamics of the city and then in Section 6 to do welfare analysis.

3.1. *Data and mapping*

As noted in Section 1 and detailed in the Data Appendix, we capture the characteristics of the built environment of Nairobi at a very fine spatial resolution. We work with $150\text{ m} \times 150\text{ m}$ grid cells, based on data aggregated from 40 cm resolution to $3\text{ m} \times 3\text{ m}$ cells and to the grid squares that we work with. From aerial photographs, we have tracings of all building footprints in 2003 and 2015 and we have LiDAR height data for 2015 (Ramani Geosystems, 2015; World Bank, 2015). LiDAR records on average 1–3 points per square meter on the ground and is accurate in height to within a meter or less. Each building's height is the average of the LiDAR recordings that intersect with the building's 2D footprint and allows for sloping roofs and differential height in building segments. To infer 2003 heights, we assume that if building footprint is unchanged between dates then so is height. For demolished buildings, we assume 2003 heights equal the average height of unchanged buildings in the eight queen neighbouring grid squares, although that is likely to overstate their height, since demolished buildings are likely less tall than unchanged ones. Each gridcell is also assigned mean elevation and ruggedness based on USGS (2005).

In Figure 3, we show two mappings of slums and define the area of the city we work with. The star in Figure 3 marks the city centre, from which all gradients we estimate emanate. This centre is the traditional central business district and is defined by the brightest nightlights pixel in Nairobi in 1992. Figure 3 shows our economic boundaries for the city, a dashed outline for 2003 and a solid one for 2015. These boundaries are based on built cover. For a $(150\text{ m} \times 150\text{ m})$ grid square to be in the city, the average roof cover in cells whose centroid is within a 900 m radius of the cell must be above 10%; and we only keep those cells that contiguously connect to the centre. To focus on private sector development, we remove all grid squares entirely in permanent public uses and amounting to 11% of land in the 2003 city boundary; neighbourhood schools and roads remain (see Supplementary Appendix S2). There is a large extensive margin expansion between 2003 and 2015 of about 60% in land area. Nairobi's sausage shape and the lack of expansion directly north and south of the centre arises because of permanent fixtures (Harari, 2020): in the south an airport and a national park and in the north a state forest.

Our analysis requires a distinction between formal and informal settlements (slums). We base our slum mapping on a 2011 classification by IPE under the Kenya Informal Settlements Improvement Program, with details given in Supplementary Appendix S2 (IPE Global Private Limited and Silverwind Consultants, 2013). The IPE mapping used satellite imagery, topographic maps, and on the ground surveying to define slums as having aspects of lack of planning and lay-out, low house quality, poor infrastructure, or insecure tenure. They also classify slums by land ownership, in particular private versus government, and some residual categories such as riparian and roadside slums, where riparian also includes large tracts of privately owned land and land under possessory rights. Supplementary Figure SA2.7 in Appendix S2 shows how ownership and slum classification varies with distance from the centre. An earlier delineation of slums comes from a 2003 land use map prepared by the CSUD at Columbia University (Williams *et al.*, 2014). While the effective definitions differ in precise detail across the two studies, we use the 2003 mapping to try to distinguish slum areas that underwent formal sector

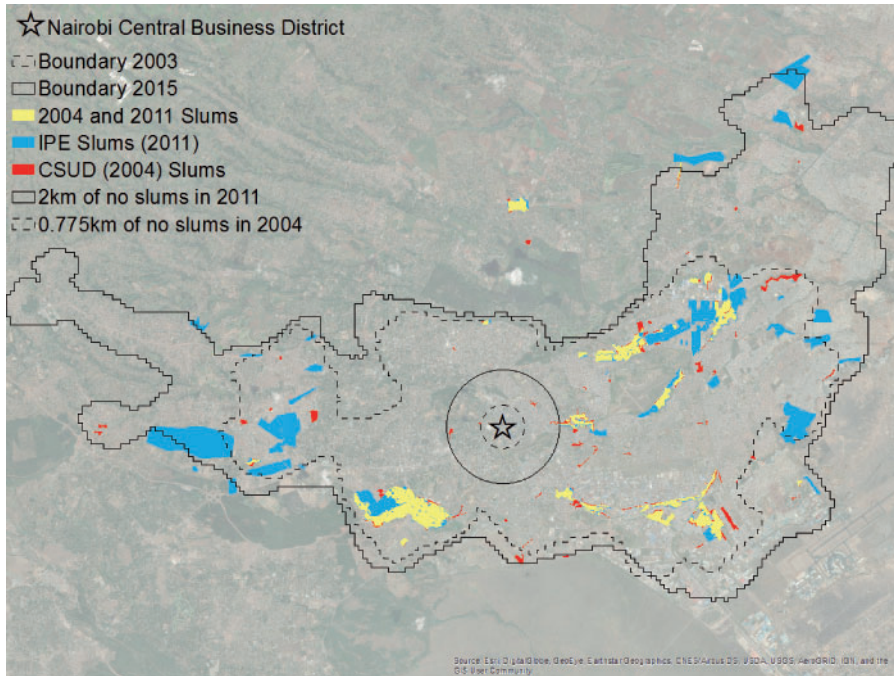


FIGURE 3

City shape and slums

Notes: This figure maps two slum definitions: IPE from 2011 (blue), CSUD from 2003 (red), and the overlap of the two (yellow). The 2015 boundary of the city is traced with a solid black line, and the 2003 boundary is traced with a dashed black line. The central business district is denoted with a star and is circled for areas without slums in 2004 (dashed circle) and in 2011 (solid circle). Satellite imagery is faded in the background to give a sense of context.

development between 2003 and 2011. While we rely on the 2011 IPE mapping to define slum areas in empirical work, the categorization of formal versus slum buildings and the drawing of slum borders can have fuzzy portions. While in Section 2.2.2, there was a clear general distinction in building materials between the sectors, there can be more of a continuum where, for example, some slum buildings are made of load bearing materials. They are just in crowded areas that are irregularly laid out.¹⁸ Any categorization has issues, and we experiment in robustness checks with other divisions.

In Figure 3, slum areas are marked yellow if recorded in both studies, blue if only in the 2011 IPE and red if only in 2003 CSUD. We will look at these red areas to show aspects of slum conversion to formal sector use. However, there seems to be little overall slum removal. Areas near the centre with no recorded slums are marked by the dashed and solid rings for respective periods. While the area with no slums expands considerably from about a 0.8 km radius to about a 2 km radius around the centre by 2011, the removed slum areas are tiny. The map suggests that most slum extensive margin expansion (blue) is near the 2003 fringe of the city and beyond. The

18. There are writings about semi-formal buildings, which are built tall of cheap materials and have a reputation in the press of collapse. This sector does not seem to be growing given the bad publicity, although semi-formal construction had a modest boom and the well-publicized collapses. There is no mapping of semi-formal areas, but based inspection of satellite images and on Huchzermeyer (2007) we think slum boundaries largely contain these structures, where the Huruma area Huchzermeyer studied is within the IPE slum mapping we use.

well-known large slum of Kibera, which we will discuss later in some detail, is the large slum area directly south-west of the centre (ranging from about 3–5 km of the centre).

Data on space-rent and housing characteristics are derived from a georeferenced household data set from the 2012 ‘Kenya: State of the Cities’ survey by the National Opinion Research Center (NORC) and we will use these to infer base formal and slum rents (Gulyani *et al.*, 2012). The NORC used the Census mapping of slums noted above but sampled within those areas to get what they thought were true slum versus formal sector properties; we accept their on-the-ground classification for the sample properties. Second, on prices, we have asking prices for vacant land listed in late 2015, obtained by scraping from Property24 (2015), a website that advertises property for sale in Kenya. All listings fall in the formal sector. In Supplementary Figure SA2.4 of the Appendix, we show a map of the locations of these properties (noting some dots overlap) and the NORC sites. Nairobi has a relatively sophisticated property market. We consider the listing services to give excellent coverage of formal sector land and there is a good scattering of all types of properties in the map.

We have made choices above as to what is the city centre, how slums are defined, and whether there is an important alternative centre. In Section 3.2.3, we perform robustness checks on our results, concerning these choices. Second, we rely on specific sources for land value and rental data. As such there is the issue of selection in the listings for land price data and in NORC surveying of slums, and the potential for omitted variables in regression analysis. We will discuss potential biases below. Third concerns what land value data reflect in markets where there can be bubbles. Supplementary Appendix Section SA2.4 analyses this issue for Nairobi, concluding that bubbles are not an issue in the data we use.

3.2. Characteristics of Nairobi’s built environment in the cross-section

Our cross-section description of the city starts with how the built fabric varies with distance from the centre, from which we derive parameters of the model and evaluate key model assumptions. As well as fitting our model, looking at variation with respect to distance from the centre is standard in the empirical literature (e.g. Ahlfeldt and McMillen, 2018; Combes *et al.*, 2019). Such regressions take the form

$$\log(y_i) = b_1 + \beta_1 x_i + \beta_c Z_i + \epsilon_i.$$

Outcomes, y_i , include land prices and measures of the built fabric in formal and slum areas. Where the dependent variable is space-rent or land price observation i denotes the household or vacant lot; where it is a feature of the built fabric observation i denotes the 150 m × 150 m grid square. x_i is distance to the city centre so that β_1 is the gradient. Following the model, but also the literature, regressions are semi-log, so a gradient coefficient can be interpreted as proportionate change per kilometre distance from the centre. b_1 is a constant term. Z_i are outcome-specific relevant control variables discussed below. For the error term, omitted variables will be a concern.

Results on estimated relationships are presented in figures and in tables. The figures for this section cover the 2003 bounded area of the city, cut at 10 km from the centre, based on observations at the 150 m × 150 m grid square level. Regressions cover all 6,470 grid squares of the 2003 city. For this section, regression results are presented in Tables 1 and 2 below where all regressions contain controls on grid square elevation and ruggedness. Regressions for prices have further controls, and tables report the gradients and an “intercept” at the city centre which is $b_1 + \beta_c \bar{Z}_i$ for Z_i evaluated usually at median values for the city. More details are in table notes and coefficients on all covariates of key regressions are in Supplementary Appendix Table SA2.5.

TABLE 1
Formal sector

	Ln formal BVAR	Ln formal CAR	Ln formal height	Ln formal redeveloped height; quantile: 80th percentile	Ln formal unchanged height; quantile: 20th percentile	Ln land price: m ² , \$2015.	Ln formal space- rent: m ³ , \$2015.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Distance to centre	-0.0493 (0.0124)	0.0239 (0.00888)	-0.0763 (0.00791)	-0.101 (0.00521)	-0.0763 (0.00223)	-0.172 (0.0476)	-0.0855 (0.031)
Intercept ($x=0$) for typical item	0.747 (0.0829)	-1.672 (0.0571)	2.487 (0.0537)	3.315 (0.0356)	2.131 (0.015)	7.254 (0.277)	3.148 (0.071)
Ruggedness & elevation	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Other controls	No	No	No	No	No	Yes	Yes
Observations	5,435	5,435	5,435	4,621	5,079	136	361
R-squared	0.035	0.043	0.203			0.292	0.244

Notes: All columns except 7 are based on 2015 data for observations inside the 2003 extent of the city. Column 7 is based on NORC data and restricted to observations inside the 2003 extent of the city. Standard errors in parentheses. Standard errors are robust and in columns 1–3 are clustered based on a 750 m × 750 m grid. Errors in cols. 4 and 5 are not clustered; in column 6 they are clustered at the neighbourhood area from the online listing service; and in column 7 at the census enumeration area. Details and full results for columns 4, 6, and 7 are in [Supplementary Table SA2.5 of the Appendix](#). Reported intercepts are predicted values at the city centre for median ruggedness and elevation for cols. 1–5. In those columns, for height regressions we only include grids for which there is cover. For BVAR and CAR regressions about 5% of observations have no cover and hence volume (e.g. playing fields, overpasses, small parks and the like). A Tobit including these as “censored” at 0 yields almost identical slope coefficients. In column 6, the reported intercept is the predicted sales price at the centre per m² for a lot based on median area, ruggedness, and elevation for the sample. In column 7, the intercept is the rent per cubic meter of space for a house with typical characteristics in the formal sector: mean values for categorical variables and median for continuous for the NORC formal sector, for the regressions where we force the gradient to have the model slope of -0.071 solved in the next section.

3.2.1. Formal sector. Intensity of land use is measured by volume of built space in cubic meters (m³) per square meter (m²) of land area, which we call the built volume to area ratio, BVAR and correspond to $v_i(x, t)$ in the theory. The gradient for BVAR is given in column 1 of Table 1 and illustrated for the raw data with a smoothed fit in Figure 4, which shows the confidence intervals and the 25th and 75th percentiles of observations.¹⁹ Formal sector BVAR near the centre is very high, averaging around 8 m³ of space per m² of ground area (where ground area includes roads and minor public spaces) in Figure 4; it declines at a rate of 4.9% per km from the centre from Table 1, column 1. The 25th and 75th percentile of observations. Note there is heterogeneity across grid squares in BVAR’s at each distance, reflecting, in part, differential timing of formal sector development of grid squares, based on differential geography and the specific histories of properties’ paths to formalization, as well features such as roads.

BVAR is the product of height and the cover-to-area (CAR, or $c_i(x, t)$ in the model), which have following gradients. In Table 1, column 3, average heights of buildings decline with distance from the centre at a rate of 7.6% per km. This compares with height gradient numbers for Chicago in the 1960s or 70s from [Ahlfeldt and McMillen \(2018\)](#).²⁰ As Figure 5 shows, there are very tall buildings at the centre in Nairobi; at 0–1 km from the centre average grid square height (based on

19. While we have the universe of observations, smoothing requires estimation to show a continuous gradient. As such, the displayed confidence intervals on the mean are extremely tight. To show the dispersion, we plot smoothed estimates of the 25th and 75th percentile of observations at each distance. Smoothing involves grid squares whose centroid is in a 300 m moving window going out from the centre.

20. The authors use different functional forms and look just at tall buildings. But at say 5 km from the centre their slope estimates are similar. However, for later years they have steeper slopes.

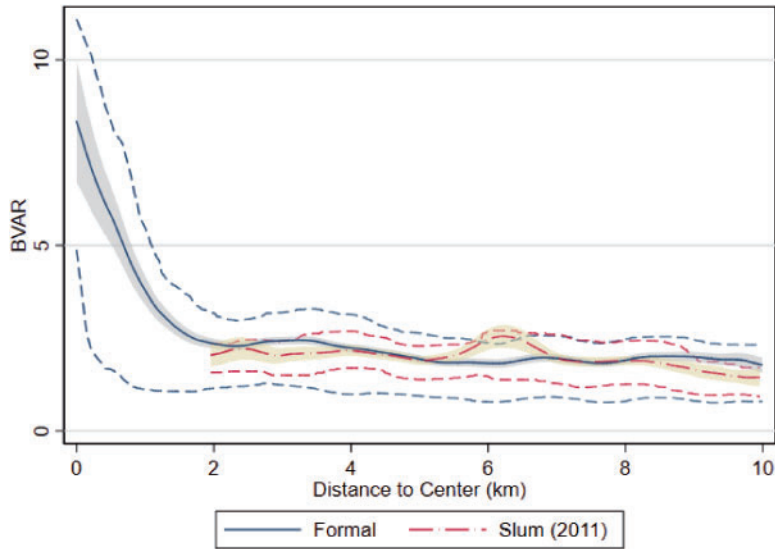


FIGURE 4

Built volume per unit area (BVAR)

Notes: Built volume to area ratio (BVAR) by distance from the centre for formal and slum sectors. Lines show the local average BVAR, shaded areas show local 95% confidence intervals, and dashed lines show local 25th and 75th percentiles. Local statistics calculated using an Epanechnikov kernel with bandwidth of 300 m.

3 m × 3 m pixel) is about 10 storeys (at 3 m a storey); the data indicate that 6% of the grid squares are over 16 storeys. For CAR, we assumed in the theory that the formal sector has constant cover to area ratio throughout the city. The gradient in Table 1, column 2 has a zero slope; and the solid blue line in Figure 5 indicates that CAR does not change much across the city and is not decreasing with distance from the centre. The decline in formal sector BVAR is thus due to height rather than cover.

To recover formal sector parameters in line with the theory, we focus on the gradient for heights of newly redeveloped buildings, as given in equation (21).²¹ We do not know building ages per se, just whether buildings are new since 2003. To pick the most recently redeveloped buildings from 2003 to 2015 interval, we first take the sample of redeveloped buildings from 2003 to 2015. Based on the theory where at any distance heights of new buildings will rise with time, we focus on the 80th percentile, balancing out wanting the most recent and hence taller buildings, against getting extreme outliers. Column 4 estimates the gradient for redeveloped buildings using a quantile regression. Fortunately, for redeveloped buildings, different quantiles, and OLS estimates of the gradient slope are very similar.²² From Table 1, column 4 the slope is -10.1% per km, noticeably larger in absolute value than for the overall stock.

What are identification issues for these regressions on the built environment in cols. 1–4? The data are for the universe within the 2003 city, so there is no sample selection. The concern would be that there are geographic conditions (affecting the D 's in the model) which could be correlated with distance to the centre and affect gradients, which is why we control for elevation and ruggedness. That said, the two key equations we use to parameterise the model, the slum

21. Note we cannot do redeveloped BVAR, since we cannot properly assign land area to new vs. old buildings.

22. Gradient coefficients on redeveloped buildings are stable, with OLS and quantile regressions up to the 90th percentile all having gradients in the neighbourhood of -0.10 . But intercepts rise, as we raise the percentile level.

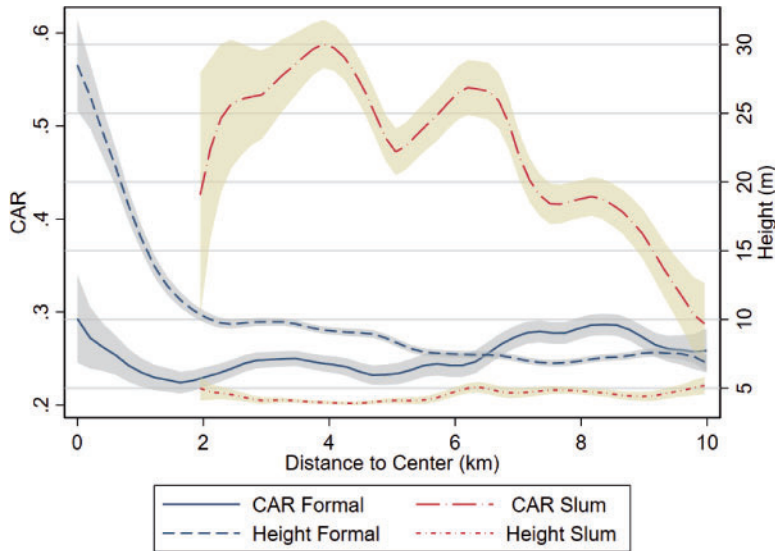


FIGURE 5

Built cover per unit area (CAR) and height

Notes: Cover to area ratio (CAR) and building height by distance from the centre for formal and slum sectors. Lines show the local average CAR and height, and shaded areas show local 95% confidence intervals. Local statistics calculated using an Epanechnikov kernel with bandwidth of 300 m.

BVAR and the formal sector 80th percentile of height quantile regression, should not be affected by D magnitudes, according to the model. Correspondingly, whether we control for ruggedness and elevation or not, the estimated coefficients are stable.

Turning to price and rent gradients, column 6 of Table 1 uses scraped land price listings from the fall 2015 [Property24 \(2015\)](#) data to estimate the formal sector land price gradient with respect to distance from the centre, with asking prices (per square meter) in USA\$. An exponential form captures the relationship well, and the gradient is steep, with price declining by 17.2% per km of distance from the centre. Such a rate of decline means land prices vary almost 6-fold from the centre out to 10 km. This slope is right in the middle of the range of land price gradient slopes across French cities in [Combes et al. \(2019\)](#), Table 3).

For column 6 of Table 1, identification issues are a concern. There are two issues, sample selection, and bias in estimates within the selected sample due to unobserved property features affecting prices that could be correlated with distance to the centre. On selection, while we believe most lots in the formal sector are listed, we do not know the universe of properties for sale. [Supplementary Figure SA2.4](#) in the [Appendix](#) shows a good scattering of listing points throughout the city. If there are unlisted properties, they may be different, on more challenged land or land with worse amenities, both of which could vary with distance from the centre. For example, if these omitted properties are first formal sector developments, we will show later that these would be on worse quality sites nearer the centre (last to be developed). In this case, selection could bias our estimate of the intercept upwards and our estimate of the slope downwards (more negative). On the issue of omitted variables, we control for plot area, ruggedness, elevation, and whether the listings have an exact street address (which many properties do not) as an indicator of a quality differential. However, unobserved amenities could be better or worse near the centre, and the direction of bias is ambiguous. In short, depending on the relative extent of

selection and omitted variable biases, our estimate of the land rent gradient could be biased either way.²³

The final column in Table 1 estimates the space-rent gradient with data from the NORC, using hedonic price regressions for space-rents. Column 7 reports gradients and intercepts in 2015 USA\$ for m³ of volume.²⁴ We will utilize the intercept only in work below; this space-rent gradient can be sensitive to specification unlike other columns. These regressions control for house characteristics in order to define rents per unit volume for a *typical* formal sector house; full hedonic regressions are reported in [Supplementary Appendix Table SA2.5](#). Table 1, column 7 gives the gradient of formal sector space-rents, which is flatter than that of land-rents as suggested by the theory. In column 7, its slope of -8.6% per km is close to what we back out of the model (-7.1%) in Section 4.1 below.

3.2.2. Slum areas. Table 2 gives gradients for the slum sector. Slum areas should exhibit a decline in volume with distance from the centre, with the decline driven by diminishing cover per unit area, not by a change in height. Table 2 and Figures 4 and 5 give results. Column 1 of Table 2 indicates BVAR declines significantly with distance from the city centre at a rate of 9.5% per km. In column 3, slum heights are flat in terms of distance from the city centre as assumed in the model and as pictured in Figure 5 by the red-dotted line. With constant slum height, gradients for slum BVAR in column 1 and slum cover to area ratio, CAR in column 2 should be the same; the estimates of -0.0948 and -0.103 in Table 2 cols. 1 and 2 confirm this.²⁵ Slum CAR near the city centre in Figure 5 is very high at 50% or so, more than twice the 25% number for the formal sector. This means that slums have little green/open space around houses, with attendant loss of amenity.²⁶

The final column of Table 2 gives the slum space-rent gradient. Note that in the model and in the data, typical formal and slum sector houses are not comparable; and we have separate gradients for each. The slum space-rent gradient is flat, in contrast to that in the formal sector. In equation (6) of the model, observed space-rents are constant throughout the city; the insignificant coefficient of -0.0094 validates the modelling that gives this result. The intercepts of the space-rent equations in the formal and slum sectors in Tables 1 and 2 are, respectively, 3.148 and 1.886. These will be used later in welfare analysis to infer land-rent differentials by sector. The formal-slum differential in space-rents reflects different amenity values, incorporated in the model by the parameters a_i .

Again, there are issues of identification for this NORC sample. On selection, [Supplementary Figure SA2.4](#) in the [Appendix](#) shows a good scattering of points throughout the city for formal and slum housing, covering newer and older slums within the 2003 city. Households were randomly sampled and stratified based on the 2009 Census definition of slums. On bias and unobserved attributes that could vary systematically with distance from the centre for developed formal sector and slum housing, we simply do not know the likely direction of bias. We also note that

23. Using reports from HassConsult, an alternative set of vacant land listing prices, the estimated gradient from regressing \ln price per square metre on distance gives a similar slope -0.198 (0.0309), within one standard error of that estimated here. See [Supplementary Appendix Section SA2.4](#) for details.

24. Calculations assume each m² of floor space yields 3.0 m³ of volume (given typical ceiling height) inflates 2012 nominal rents at 8% a year based on reports from <http://www.hassconsult.co.ke>, converts monthly to annual rents and Kenyan shillings at the 2015 exchange rate of 100 KS to a USA\$.

25. Note the slopes for slum CAR and height ($-0.103 + 0.0107$) sum to almost that for BVAR, -0.0948 .

26. We also have paved road surface from high-resolution SPOT satellite data for 2015. Formal sector roads are about 15% of area near the centre while in slums they maybe hit 5%.

TABLE 2
Slum areas

	Ln slum BVAR	Ln slum CAR	Ln slum height	Ln slum space-rent m ³ , \$2015.
	(1)	(2)	(3)	(4)
Distance to centre	-0.0948 (0.0172)	-0.103 (0.0171)	0.0107 (0.00871)	-0.0094 (0.0243)
Intercept ($\alpha=0$) for typical item	1.275 (0.1176)	-0.116 (0.1049)	1.375 (0.0686)	1.886 (0.0581)
Ruggedness & elevation	Yes	Yes	Yes	Yes
Other controls	No	No	No	Yes
Observations	958	958	958	439
R-squared	0.104	0.142	0.017	0.391

Notes: All columns are based on 2015 data for observations inside the 2003 extent of the city. Robust standard errors are in parentheses. Errors in columns 1–3 are clustered based on a 750 m × 750 m grid and in column 4 are clustered at the enumeration area. Details and full results are in [Supplementary Table SA2.5](#) of the [Appendix](#). Reported intercepts are predicted values at the city centre for median ruggedness and elevation for cols. 1–3. In column 4, the intercept is the rent per m³ of space for a house with typical characteristics in the slum sector: mean values for categorical variables and median for continuous in the NORC slum sector for a regression where we force the gradient to have the model slope of 0.

features like ruggedness and elevation on average are the same across sectors with insignificant differences.²⁷

A further point of interest in [Figure 4](#) is that, at 2 km and beyond, slums and the formal sector deliver generally the same BVAR. For the opposing views of whether formal sector height trumps slum coverage in providing volume of built space per unit land, in Nairobi, they do equally well on average, albeit at different quality levels. However, while each sector has similar BVAR, slums occupy a smaller fraction of the city’s surface area and hence provide just 10% of total building volume.

3.2.3. Robustness. We perform robustness checks on various definitions or classifications, we imposed on the data. The first concerns the city centre being chosen as the brightest nightlights’ pixel in Nairobi in 1992. Alternatively, using the current nightlights from NASA’s Visible Infrared Imaging Radiometer Suite to identify the top 1% of grid-cells readings in Nairobi, we can define the centre as the brightest pixel in the largest cluster of these top 1% pixels ([Supplementary Appendix Section SA2.4](#)). That moves the star in [Figure 3](#) by about 400 m. The second check concerns the presence of a second major centre in Nairobi around which gradients might orient. The most likely candidate is a large industrial area to the east and somewhat south of the centre. We experiment with specifying this as a second centre influencing or even potentially dominating the gradient from the city centre that we estimate. Finally, there is the definition of slums. Alternative to IPE in 2011 is a 2009 Census map of slums classifying census tracts as slums if they are “unplanned”; this results in a looser definition of slums and larger tracts, as can be seen in [Supplementary Figure SA2.3](#) in the [Appendix \(Kenya, 2009\)](#). We will experiment with using this definition of slums.

We report robustness results for the five equations that we use below to derive parameters of the model: the land, slum rent and formal rent price equations and the equations for slum BVAR and formal sector redeveloped height. For the slum BVAR and formal sector height equations,

27. Mean [SD] log elevation in the slum and formal sector are 7.43 [0.045] and 7.40 [0.028] and log ruggedness are 1.58 [0.54] and 1.43 [0.57], respectively.

these involve adding a control for distance to the main industrial centre to the base equations, using the alternative centre point based on recent (not 1992) nightlights in the equations, and using the Kenya (2009) Census definition of slum areas (as unplanned settlements and having density over 3000 per sq. km). For the price gradients, we examine the impact of having a differently placed city centre and having a second centre. We do not change slum area definitions for the price equations. Land listings cover only the formal sector; and we accept that, in their sampling, NORC have correctly defined on the ground allocations of households to slum and formal sector environments. The results are reported in [Supplementary Table SA2.6](#) of the [Appendix](#). The new city centre definition has a negligible impact on all estimated gradients and intercepts for the five equations. Adding distance to the industrial centre to the base specifications adds a covariate that is always insignificant, has a small impact on all intercepts, and only has a noticeable effect on the CBD distance gradient in one case, formal sector rents. However, we do not use the gradient for that outcome to solve the model parameters below, only the intercept which is minimally affected. Finally, using the census definition of slums similarly has a negligible impact on gradients and intercepts for slum BVAR and redeveloped heights.

4. THE DATA AND THE MODEL

We now derive parameters of the model, using 2015 cross-section information from [Tables 1](#) and [2](#) and relationships in the model. The parameter estimates are of intrinsic interest, and we compare them with other estimates in the literature, where available. In [Section 5](#), we use the parameterized model to predict dynamic changes between 2003 and 2015 and compare these with what we find in the data.

4.1. *Solving for the cross-section built environment*

The model, in cross-section, is fully determined by parameters describing technology and amenity, $\gamma, \alpha, \kappa_I, a_I$, and (κ_F/Φ) , together with knowledge of space-rent at the centre and its gradient, $p(0, 0)$ and θ . (Formal sector choices depend on κ_F/Φ ; here we calculate the ratio, and in [Section 4.3](#) separate the components.) Relevant values and model equations are set out in [Table 3](#) which divides into two parts, the upper block giving gradients of variables with respect to distance from the centre and the lower the levels of parameters and variables at the centre, $x=0$ (i.e. intercepts reported in [Tables 1](#) and [2](#)).

Looking first at [Table 3](#) panel a; for the formal sector, we use gradients on newly developed buildings, taking the distance coefficient of new building height ([Table 1](#), column 4) as the measure of how volume varies with distance, and similarly, land values as the measure of $R_F(x, \tau_i)$, the present value of land-rent that accrues over the life of a structure. For slums, the volume measure is BVAR; there is no land value measure available since slum land is not widely transacted. The three equations in this part of the table give values for parameters γ, α and θ .

[Table 3](#) panel b equates intercepts from the regressions with expressions for the level of variables from the theory. These parameters play a role in model simulations where we need levels. For formal sector volume of new buildings, we use height times the median cover of 0.247 throughout the city. This is in preference to the intercept in column 2 of [Table 1](#) (which gives a CAR of 0.188) but has low explanatory power given the undulating but basically flat relationship for CAR in [Figure 6](#) below. For slums with their malleable capital all that matters is BVAR. The final row of the table uses the space-rent data and together with values of γ and α , they give values of $\kappa_I, a_I, (\kappa_F/\Phi)$ and $p(0, 2015)$.

TABLE 3
Model parameters

a. Model parameters from gradients		
Gradients:	Formal:	Slum:
Volume m ³ ,	$\frac{dv_F(x, \tau_t)}{dx} \frac{1}{v_F} = \frac{-\theta}{\gamma-1} = -0.101$	$\frac{dv_I(x, t)}{dx} \frac{1}{v_I} = \frac{-\theta\alpha}{\alpha-1} = -0.0948$
per m ² land.	Equation (21); Table 1, column 4	Equation (4); Table 2, column 1
Land-price,	$\frac{dR_F(x, \tau_t)}{dx} \frac{1}{R_F} = \frac{-\theta\gamma}{\gamma-1} = -0.172$	
per m ² land.	Equation (21); Table 1, column 6	
Solutions: $\gamma = 1.703, \alpha = 3.983, \theta = 0.071$		
b. Levels of variables and further parameters		
Volume m ³ ,	$v_F(x, \tau_t) = \left[\frac{p(x, \tau_t)}{\gamma} \frac{\Phi}{\kappa_F} \right]^{1/(\gamma-1)} = 6.80$	$v_I(x, t) = [a_I p(x, t) / \kappa_I \alpha]^{\alpha/(\alpha-1)} = 3.58$
per m ² land.	$= \exp(3.315) \times 0.247$	$= \exp(1.275)$
	Equation (10a); Table 1, column 4 for height, times median formal sector CAR = 0.247	Equation (4); Table 2, column 1
Space-rent, per m ³ volume.	$p(x, t) = 23.29 = \exp(3.148)$	$p(x, t) a(v_I(x, t)) = \kappa_I \alpha = 6.59$ $= \exp(1.886)$
	Table 1, column 7	Equation (6); Table 2, column 4
Solutions: $p(x, t) = 23.29, a_I = 0.734, \kappa_F / \Phi = 3.554$ Levels evaluated at $x = 0, t = \tau_t = 2015$, in \$US		

4.2. Discussion

Technology and amenity parameters, γ and α , come from cross-section gradients and give diminishing returns in formal sector construction costs and slum crowding, respectively. They also imply shares of land-rent and construction cost in spending on housing. The value of $\gamma = 1.70$ implies that the share of land-rent in formal sector revenue, $1 - 1/\gamma$, is 0.41, while $\alpha = 3.98$ implies a corresponding share in the informal sector, $1 - 1/\alpha$, of 0.75. The formal sector share of 0.41 is similar to that in Case (2007) for the USA but higher than the 0.30 for Paris in Combes et al. (2017). For slums, we know of no data to make comparisons, since typically slum lands are not transacted privately. However, given the low construction costs of slum housing, such a high land-rent share seems reasonable. Note that $\alpha > \gamma$ is the condition for the model to predict that slum land-use at the city edge precedes formal development.

Land-rent is the product of space-rent per unit volume (\$ per m³), space per unit land (BVAR m³ per m²) and the share of space-rent that is attributable to land, $1 - 1/\gamma$ in the formal sector and $1 - 1/\alpha$ in slums. We find that, at the centre in 2015, $x = 0$ and $t = 2015$; $r_F(x, t, t) = \$65.38$ and $r_I(x, t) = \$17.67$ (using equations (5) and (12) with numbers from Table 3). Moving away from the centre these imputed land-rents drop at respective proportional rates $\theta\gamma/(\gamma - 1) = 0.17$ per km and $\theta\alpha/(\alpha - 1) = 0.095$ per km, nearly twice as fast in the formal sector as the informal. The wide gap in land-rents near the centre is indicative of inefficient land-use (an argument that we will develop precisely in Section 6), although we note that from equations (15) and (16), efficient switch points are generally not at the point of equality of r_F and r_I .

What are the components of these changing land-rents? In the formal sector, the distance gradient is driven principally by commuting costs reducing net wages and hence space-rents (as given by equation (1)); land-rents fall faster than space-rents because of declining volume per unit land. For slums, as we saw above, observed space-rent is constant as lower commuting costs

near the city centre are offset by greater crowding; the decline in land-rents is therefore due to lower volumes.

There are two other predictions of interest, with details in [Supplementary Appendix Section SA1.6](#). The amenity derived from a unit volume of slum housing at the centre is 28% of a unit of formal volume; slum crowding decreases with distance, so this fraction rises, reaching 58% at 10 km.²⁸ This change reflects the quality continuum aspect of slums. The construction cost of a unit volume of slum housing is 12% that of formal sector construction cost per unit volume at the centre (cost expressed on an annual amortization basis); formal construction costs per unit volume decrease with distance as buildings get less tall, so at 10 km this fraction reaches 25%. As implied, formal sector unit construction costs per unit volume at 10 km are 50% of what they are at the centre.

4.3. Solving for the dynamic environment

The city is characterized by changing land-use and periodic redevelopment, as is modelled in the theory. Quantifying this requires two further pieces of information. One is the rate of interest, ρ , and we use the real interest rate $\rho = 0.057$, which is the average of the World Bank real interest rate (lending rate adjusted for inflation) for Kenya for the 14 years post 2002.²⁹ The other is the expected rate of increase of space-rent, \hat{p} . We have estimates of formal sector land-rents, $r_F(x, t, t) = \$65.38$ ($x = 0, t = 2015$, previous sub-section) and the capital value of land at date of redevelopment, $PV_F(x, \tau_i) = \$1414$ ($= \exp(7.254)$, Table 1, column 6). Equation (19) links this capital value to the level and rate of increase of flow rents, dependent on price growth, and hence enables us to back-out the expected price increase. Using these numbers and the value for γ from Table 3, we solve for Φ , $\Delta\tau$, and \hat{p} in equations (17–19), restated here as:

$$\Phi = \int_0^{\Delta\tau} e^{(\hat{p}-\rho)t} dt = \frac{1 - e^{(\hat{p}-\rho)\Delta\tau}}{\rho - \hat{p}}, \quad \Delta\tau = \frac{\gamma - 1}{\hat{p}} \ln \left[\frac{\gamma}{\gamma - \rho\Phi} \right],$$

$$PV_F(x, \tau_i) = r_F(x, \tau_i, \tau_i) \Phi / \left[1 - e^{-\left(\rho - \frac{\hat{p}\gamma}{\gamma - 1}\right)\Delta\tau} \right].$$

Solving these equations gives $\hat{p} = 0.0092$, $\Phi = 20.64$, and $\Delta\tau = 89.29$ years. Thus, real space-rent appreciation is estimated to be about 1% a year. This is broadly consistent with the space-rent series produced by HassConsult as described in [Supplementary Appendix Section SA2.4](#). The value-to-rent ratio, $\Phi = 20.64$, is close to that implied by an infinite stream discounted at 5.7%, and is in the middle of range of ratios reported on the internet by realtors for US cities.³⁰ The length of life of new buildings, 89 years, is novel and an estimate for which we know of no easy comparison.

5. CITY DYNAMICS: THE DATA AND THE MODEL

As reported in Section 3.1, we have tracings of all building footprints for 2003 and 2015, and LiDAR height data for 2015. In this section, we use these data to describe the patterns of change that occurred in the city, and then compare these actual changes with predictions of the model. As we have seen, the model is calibrated on 2015 cross-section data, so using the model to predict the timing and extent of changes that occurred in the city is a demanding comparison.

28. The amenity parameters $a_I = 0.734$ and $a_F = 1$ are constant.

29. <https://data.worldbank.org/indicator/FR.INR.RINR?locations=K>

30. For example, <https://smartasset.com/mortgage/price-to-rent-ratio-in-us-cities>

TABLE 4
Dates of development

	Distance from centre, x						
	0.5 km	1.5 km	2.5 km	3.5 km	10.5 km	12.5 km	14.5 km
τ_1 : Formalization							
$D(x)=0$	1917	1924	1932	1940	1994	2009	2025
τ_2 : First redevelopment							
$D(x)=0$	2006	2014	2022	2029	2083	2099	2114

Model prediction with space-rent path: $p(x, t) = p(0, 2015) \exp(\hat{p}(t - 2015) - \theta x) = 23.29 \exp(0.0092(t - 2015) - 0.071x)$

5.1. *Development and redevelopment in the formal sector*

We look first at the formal sector, where the model predicts that there is a wave of development, followed by a wave of redevelopment (Section 2.5 and Figure 2). Figure 6 is the empirical counterpart, giving formal sector volume change (2003–15) per unit area at each distance and its decomposition into two parts, infill and redevelopment out to 12 km of the 2015 city. Two distinct waves of change are apparent. The wave further out, peaking at around 9.5–10 km and significantly higher than other rates, is largely infill, defined as new buildings not intersecting any 2003 buildings. This is the empirical counterpart of formalization; it is construction of formal structures at the edge of the city, some of them on land temporarily occupied by low-density slums, and some on land previously in non-built use.³¹ The wave nearer the city centre peaks at 3–4 km from the centre and is principally redevelopment, defined as new buildings that overlap the footprint of 2003 buildings, where redevelopment volume is net of what was demolished.

These two waves are as predicted by the model. How do they line up in terms of exact timing and location? Table 4 gives timings of formalization and first redevelopment predicted by the model using values of parameters that are reported in the text above. The first row gives date of formalization, and the model predicts from equation (15a) that, if $D(x)=0$, the 2003–15 peak would occur at around 12–12.5 km; the data in Figure 6 have peak at 9.5–10 km, i.e. the model is predicting formalization slightly early. This is consistent with generally positive values of $D(x)$ that delay formalization. The predicted date of first redevelopment in Table 4 at 3.5 km is about 2030, while the distance band predicted to redevelop during the period 2003–15 is around 0.5–1 km from the CBD (Table 4, second row). The model is therefore predicting first formal development later than the wave that occurs in the data. The discrepancy arises as the predicted distance between places experiencing formalization and first development is around 11.5 km (89 years times \hat{p}/θ), whereas the peaks in Figure 6 are just 7 km apart. The “late” first redevelopment predicted by the model could—amongst other things—be attributable to using the World Bank discount rate of 5.7%; a higher rate reduces the time interval between developments, $\Delta\tau$, and hence also the distance between successive building peaks. Notwithstanding these remarks, we note that Table 4 gives first formalization in the city at 1917, surprisingly close to its date of founding, 1899. Of

31. This formal sector development near the city edge occurs as the fraction of land covered by formal buildings increases between 2003 and 2015, as discussed in Section 2.5. Not all of this land may have been occupied by slums prior to formal development. However, since we rely on one slum map in 2011, we do not know the formal status of all of this land in 2003. Costs of evicting slum tenants may also inhibit landowners from having land in slum usage close to the time of first formal sector development. Note also that “infill” can be on developer assembled lots where vacant land is built on and formally covered land is left as green space, so it is not truly infill but a rearranging of how land is used on assembled lots. There is also a small residual category (not shown) of torn down buildings with no new footprint appearing (yet) over the old.

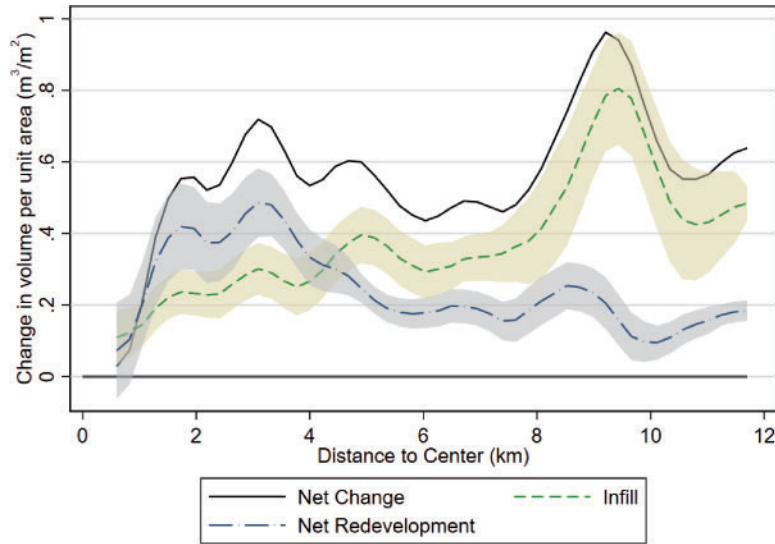


FIGURE 6

Formal sector changes in volume per unit area (2015 city to 12 km)

Notes: The change in formal sector volume, by redevelopment, infill and total net change, calculated in bins of 300 m inside the 2015 city boundary and divided by the area of each bin. Shaded areas show local 95% confidence intervals for infill and redevelopment only. Local statistics smoothed using an Epanechnikov kernel with bandwidth of 300 m.

course, such long-run projections are way out of sample and highly speculative, given historical changes through that time period.

A critical aspect of redevelopment in the data and in the model is the change in building height that it brings about. Figure 7 splits out the formal sector height gradient in Figure 5 to show how the mean heights of redeveloped buildings compare with those that did not change. In the formal sector beyond 1.5 km redeveloped buildings are significantly higher, nearly double the height of unchanged buildings in the interval 2–4 km. To get a number conceptually comparable to the model, we compare the height of the 80th percentile of redeveloped buildings, mimicking heights of newly completed buildings, to the 20th percentile of unchanged buildings, mimicking the height of those *just about* to be torn down. This is graphed in the figure and regressions on these slopes are reported in Table 1, cols. 4 and 5. From 1 to 4.5 km, the regression (using intercepts and slopes) and the figure both suggest that building height increases around 2.75–3 times with redevelopment. This compares with a model prediction given by equation (18) where volume increases by a factor of 3.23, at each redevelopment.

5.1.1. Role of terrain. An element of $D(x)$ which can be observed and that bears on the timing of development (Section 2.5) is the terrain. In [Supplementary Appendix S3](#) (Land Quality), we report the results of investigating this with water (river, wetland, pond) and ruggedness as independent variables at various spatial scales.³² There are two main findings. Figure 6 indicates infill occurring at all distances, including close to the centre. [Supplementary Table SA3.1](#) in the [Appendix](#) shows that formal sector infill near the centre is on much worse quality land than already developed buildings, indicating delay in the date in which poor terrain land is first developed.

32. Ruggedness is defined as the standard deviation of elevation across the 30×30 m grid squares of the 90×90 m square neighbourhood which is queen neighbours plus the own square.

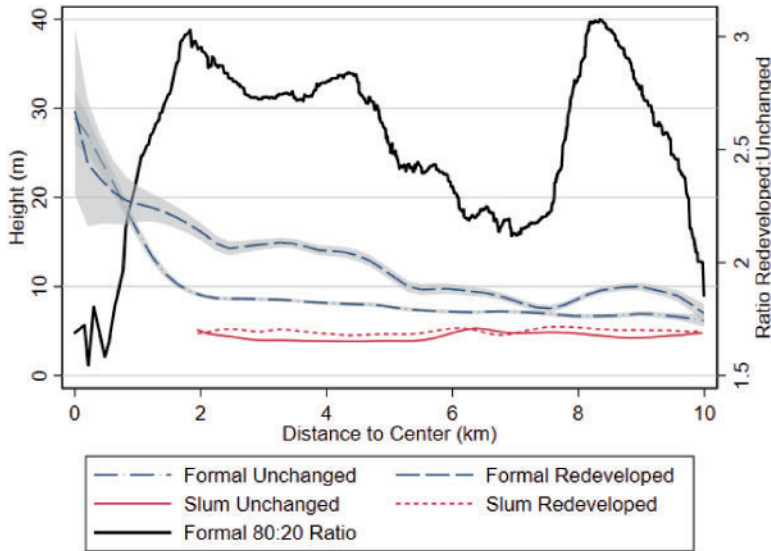


FIGURE 7

Mean height—unchanged and redeveloped buildings

Notes: Height of unchanged and redeveloped buildings by distance from the centre for formal and slum sectors. Lines show local average height and shaded areas show local 95% confidence intervals for the formal sector only. The right axis is the ratio of the local 80th percentile of redeveloped height to the local 20th percentile of unchanged height. Local statistics calculated using an Epanechnikov kernel with bandwidth of 300 m.

The effect falls rapidly with distance so, by 10 km out, there is no clear difference between infill and already developed areas. Second, [Supplementary Table SA3.2](#) shows that the height of redeveloped buildings is unaffected by underlying terrain fundamentals. Overcoming terrain involves a sunk cost of land improvement that does not affect the height of structures that are then constructed (i.e. κ_F in equation (10) is not affected).

5.2. Slum dynamics

The model predicts that, if the condition given in proposition 2 (i) is satisfied, new slums will form on the edge of the city. That near the city edge there will be conversion of existing slums into formal sector use. That existing slums will become increasingly crowded. And that there will be no slums remaining near the city centre unless there are substantial formalization costs, $D(x) > 0$.

The data are broadly consistent with these predictions. First, there are new slums. We have a proxy measure, which are areas identified as slums in the 2011 mapping but not in the 2003 mapping. As noted above, some of the differences are due to different methodologies employed by researchers in defining slums. Nevertheless, in the blue areas in Figure 3, we see large tracts of new slum development at and beyond the 2003 city edge. Referring back to dates of first formalization in Table 4, we see that in the early 2000s these should be at some distance beyond 10 km (i.e. beyond places formalizing). In Figure 8, we show the change in slum volume for slums as mapped in 2011 and the figure indicates that at about 8 km there is one peak of infill (new slum cover) and then another at 10.5 km. The difference between these two peaks is interesting. The peak at 8 km is on government owned slum lands where we presume D 's are too high to allow formal use yet and slum usage is intensified as prices rise. The peak at 10.5 km is disproportionately on private

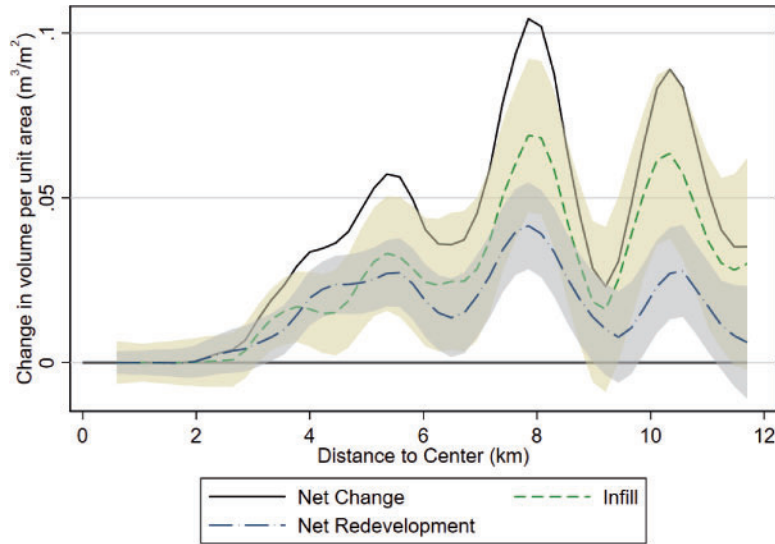


FIGURE 8

Slum sector changes in volume per unit area (2015 city out to 12 km)

Notes: The change in slum sector volume, by redevelopment, infill and total net change, calculated in bins of 300 m inside the 2015 city boundary and divided by the area of each bin. Shaded areas show local 95% confidence intervals for infill and redevelopment only. Local statistics smoothed using an Epanechnikov kernel with bandwidth of 300 m.

land, so it is on ‘market driven’ slum expansion. [Supplementary Figure SA2.7](#) in the [Appendix](#) displays these ownership patterns.³³

Second, there are some slum conversions we might infer from the 2003 and 2011 mapping of slums. In [Supplementary Appendix Figure SA2.5](#), we show rates of building teardown for 2011 slums (non-converted) and slums that disappear between the 2003 and 2011 mappings (converted). Teardown rates for these converted slums average nearly 60%, three times greater than in non-converted areas. [Supplementary Appendix Figure SA2.6](#) also shows that buildings on converted slum lands are generally built significantly taller than those in current slums.

Third, there is densification of existing 2011 slums within the 2003 city-boundary. In [Figure 8](#), we show total slum volume changes and their decomposition into “redevelopment” and infill. Up to about 6 km, “redevelopment” is akin to rearrangement of the Meccano parts and is conceptually not really distinguishable from infill. Both are part of a process of increasing crowding in slums, as CAR increases. [Figures 5](#) and [7](#) indicate, as assumed in the model, that there is essentially no increase in slum height, which remains low and uniform across the city. Volume changes are driven entirely by CAR changes.

Finally, no slums get created near the centre, but there is persistence of some large slum areas, as we discuss in detail in [Section 6](#).

5.3. Total volume and population

What are the implications of these changes for aggregate building stock and population? We have data on each of these for 2003 to 2015. As noted in [Section 1](#), total built volume within the 2003

33. In addition to purely private, we are including some riparian lands as graphed in [Supplementary Appendix Figure SA2.7](#). From [Kenya \(2018\)](#), we know these areas contain private developments, as well as areas with possessory rights.

boundary increased by 47% between 2003 and 2015, and that in the 2015 boundary by 60%. Based on the Kenya (1999) and Kenya (2009) censuses for the 2003 area of the city, population grew at 3.8 p.a. which, for the period 2003–15, would project to 57% for 2003 boundary and 68% for the 2015 city boundary

In the model, volume comes from integrating over all x at a particular date, weighting volume per unit area by area at the place.³⁴ For population, volume at each space is divided by each household's demand for space, $s_i(x, t)$, $i = I, F$, so takes the form

$$L(t) = \sum_{i=1}^{imax(t)} \int_{x_{i+1}(t)}^{x_i(t)} n(x)v_F(x, \tau_i)/s_F(x, t)dx + \int_{x_1(t)}^{x_0(t)} n(x)v_I(x, t)/s_I(x, t)dx.$$

In this expression, $n(x)$ is land area (in a circular city, proportional to distance x from the centre). The first term integrates over land in its i -th generation of development at date t (i.e. land in interval $(x_{i+1}(t), x_i(t))$), and sums over generations up to that which has been redeveloped the most times, denoted $imax(t)$. Applying this equation to volume (i.e. setting $s_i(x, t) = 1$) gives the volume increase along a ray ($n(x) = 1$) over the 12-year period of 38%. Population increase exceeds volume as rising income and space-rents create income and substitution effects in household demand for space, with the latter dominating. This gives a population increase along a ray of 56%. In a circular city each of the rates of increase are lower, at 27% and 34%; this is because the model predicts that redevelopment during this period is close to the centre than it is as noted above, i.e. covering a small land area.

To conclude this section, we remind the reader that parameters of the model were estimated entirely on cross-section data, together with an imported value of the discount rate. The model does well in predicting many aspects of Nairobi's development through time, albeit with the proviso that the long time period between developments has the effect of making the model predict redevelopment later than is in the data.

6. THE COST OF DELAYED FORMAL SECTOR DEVELOPMENT

Finally, we turn to quantifying some of the costs of persistent slums in central parts of Nairobi. Chief amongst these is Kibera, often described as the largest slum in Africa. The costs and benefits of such slums—and policy towards them—is a complex and contentious issue. We do not seek to quantify all elements (such as community dislocation), and focus on a single element of the equation, namely the real income loss due to inefficient land-use. This is measured by the loss in land values associated with informal settlement on high opportunity cost land.

6.1. Formalization costs

Nairobi was founded in 1899 and the British colonial government, as was typical throughout Africa, housed the African population in informal settlements without land title (Olima, 2001). After independence in 1963, a series of reforms resulted in over 85% of land in Nairobi becoming privately owned under charges of widespread corruption (Southall, 2005). However, older slums mostly within 6 km of the centre remain 'government owned'. These areas are not managed by the government but rather by slumlords who operate 'illegally' and make high profits. Gulyani and Talukdar (2008) estimate payback periods on an investment in a single room of

34. This is integrating under the surface of Figure 2 (anti-logged) at a particular place, and weighting by the area of each place x .

just 20.4 months. This is consistent with the fact that land is “free” to slumlords; and, by our calibration, land’s share in revenue in the informal sector is 75%. Moreover, slumlords have characteristics that are problematic for formalization. In Kibera, for example, of 120 slumlords surveyed, 41% were government officials, 16% (often the biggest holders) were politicians, and 42% were other absentee owners (Syagga et al., 2002 as cited in Gulyani and Talukdar, 2008). The political economy issue is that if the government were to auction the land it “owns” (or give it to the tenants), the slumlords would lose their claim to the revenue. Having well-connected bureaucrats and political figures opposed to formalization presents a political problem.

This problem is magnified when there are historical private claims to the land, as is typically the case. Kibera gives a nice example. The 1000 acres in Kibera were awarded to Nubian soldiers for service in 1912 by the British. They occupied a portion of the land, but at independence their claims (but not tenancy) were revoked and land reverted to the government. The majority area of present day Kibera which is not occupied by Nubians and their descendants was settled by others, on the basis of claims illegally allocated by local chiefs and bureaucrats. The moral right of the Nubian descendants to at least the land they occupy is well recognized but the unwillingness to grant them title has historically been a road-block to redevelopment (Etherton, 1971; Joireman and Vanderpoel, 2011). The literature has similar stories for other major slums in Nairobi.³⁵

6.2. *The loss of land value due to slum persistence*

In our open city model, the welfare costs of inefficient land use are given by the present value of land-rents foregone. To assess the cost of delayed formalization, we use the model and estimated parameters to calculate the present value of land-rents earned by land in different uses. The gap between the values of land in formal versus slum use measures the real income loss due to inefficient land-use, although it does not capture other costs and benefits outside the model, such as social costs of disruption involved in slum redevelopment and community dislocation, or possible productivity benefits of spatial reorganization of the city.

For a unit of land at place x , we calculate the present value, in 2015, of being held in slum use until conversion to formal use (with $D(x) = 0$) at date z , and denote this value $PV(x, z)$. For perpetual delay versus conversion in 2015 the expressions are, respectively

$$\begin{aligned} PV(x, z = \infty) &= \int_{2015}^{\infty} r_I(x, t) e^{-\rho(t-2015)} dt \\ &= \int_{2015}^{\infty} r_I(x, 2015) e^{[\hat{\rho}\alpha/(\alpha-1) - \rho](t-2015)} dt \end{aligned} \quad (22)$$

$$\begin{aligned} PV(x, z = 2015) &= \sum_{i=0}^{\infty} R_F(x, z + i\Delta\tau) e^{-\rho i\Delta\tau} \\ &= r_F(x, 2015, 2015) \Phi / \left[1 - e^{-\left(\rho - \frac{\hat{\rho}\gamma}{\gamma-1}\right)\Delta\tau} \right]. \end{aligned} \quad (23)$$

35. For example, Mathare 3 km northeast of the centre was originally a stone quarry. When the quarry closed the land went to the Department of Defence and the area was occupied by squatters. There then followed a long history of squatters attempting to set up collectives to “buy” the land in competition with land-buying companies, dissolution of the cooperatives, allegations of corruption, and competing claims on the land (Medard, 2006). Today the majority of the slum part of Mathare (about 1 km²) is private, but significant portions remain under government ownership of some sort (police, central government, Nairobi City Council). Syagga (2011) analyses how Kenyan tenure legalization can take decades to implement due to the need to reconcile interests of stakeholders and offers more examples such as the Korogocho slum.

TABLE 5
The value of land formalized at different dates

Present values in 2015 in \$2015 per m ² .							
Date of formalization, z	Distance from centre, x						
	0–1 km	1–2 km	2–3 km	3–4 km	4–5 km	5–6 km	6–7 km
$PV(x, z=2015)$	1,297	1,092	919	774	652	549	462
$PV(x, z=\infty)$	377	343	312	284	258	235	213
Slum land, km ² , 2011	0	0.0024	0.24	1.07	2.22	1.9	1.32
No. slum households, 2009	0	0	2,920	29,070	45,810	33,100	28,390
Lower bound on D (15a)	493	395	314	248	194	150	113

These present values are given in Table 5 as a function of distance from the centre, x . The first row gives the present value if formalization occurs at date $z=2015$, and the second row gives the present value if it never occurs. For each distance reported in the table, 2015 is beyond the efficient date for formalization given in Table 4. We compare the cost of never formalizing with formalizing in 2015 for each distance, $PV(x, z=2015) - PV(x, z=\infty)$. These range from \$920 per m² in the 0–1 km distance band to \$249 per m² at 6–7 km from the centre.

We illustrate the gains from conversion by looking at lands 3–4 km from the centre, which includes some parts of Kibera. At 3–4 km the cost of perpetual informality as compared to switching to formal sector use in 2015 is row 1 minus row 2, or \$774–\$284 = \$490 per m². There are 1.07 mn m² of slum land in that distance band. There is thus an aggregate gain from converting at 2015 compared to perpetual delay of \$525 mn. We give three perspectives on this.

First, with so much money on the table, why is land at 3–4 km not converting from slum to formal use at a faster rate? We have argued that there are financial costs and political barriers to formalization, and a lower bound on the monetary value of these costs is given the bottom row of Table 5. This number is per m², and is derived by solving equation (15a) as an equality. At 3–4 km this is \$248, equal to 32% of the present (market) value of land in formal use and 87% of the value in slum use, clearly non-trivial amounts consistent with our political story on Kibera. If there were slums in the centre the lower bound on D would be \$493 but, since there are none, we know that the pressure to redevelop was so intense as to overcome formalization barriers.

Second in terms of the one-off gain in land values from conversion at 3–4 km, the surplus of \$525 mn is about 10% of Nairobi's GDP in 2015.³⁶ If we add all the net gains out to 7 km, that is about \$2.5bn or 45% of Nairobi's GDP.

The third perspective hints at a political solution. Suppose slumlords were fully compensated for conversion by \$284 per m², as if they had the right to utilize the land forever. That still leaves a remaining surplus of \$490 per m² at 3–4 km. For the 29,070 households affected, the gain is about \$18,000 per household. This is a very large sum, for households paying on average less than \$700 a year in rent (from the NORC data and consistent with the rent calculations we use in Section 3.2.2). At 4–5 km, a distance including much more of Kibera, the same type of calculation gives a surplus of about \$19,000 each for the 46,000 households.³⁷

In these calculations, we have ignored moving costs for slum residents, broadly defined. For those forced to move there could be losses in terms of job location and social networks, although proper relocation programmes could mitigate those. Nevertheless, a key point of the calculations is that, relative to income and space-rents paid, there is an enormous surplus to play with to

36. GDP per capita in Kenya was \$1,090 in 2015. We set it at \$1,300 in Nairobi for a population of 4.2 mn.

37. These model-based numbers are substantially higher than preliminary back-of-the-envelope type calculations based on raw data reported in CEPR DP 11211 which give a gain of \$13,000 US per slum household for the core city.

compensate residents. In one version of a “just” non-political world this would be solved by giving tenants the land titles and allowing them to sell themselves when and if they are ready.

These calculations are subject to bias. First, to be valid, slum lands near the centre must be the same quality as available formal sector lands. That is, it is slum history and formalization costs driving the current delay in development of slum lands, not geography. The basis of comparison is the vacant land for which we have sales price data. In [Supplementary Appendix Table SA3.3](#), we perform a border experiment at slum boundaries comparing quality of slum lands with formal sector infill (vacant) lands that have just been built upon. We show that at distances out to 6 km (where these old slums are), for elevation, ruggedness in the small, ruggedness in the large, presence of water, and being lower than mean neighbourhood elevation, of the twenty cases for slums compared to formal sector, seventeen show no differential, two are better, and one worse. It seems differentials in geography do not drive non-conversion of slums near the city centre.

Second, as a downward bias, we have used formal sector residential space-rents as the basis for gain. For slum lands some highest and best use might be commercial which could have higher values nearer the city centre. Finally, as an upward bias, there is the general versus partial equilibrium distinction, in a context with heterogeneous households. In our context, with identical households and an open city, we have captured welfare gains in terms of increased land values. With households heterogeneous in terms of preferences for x 's and a 's we are effectively looking at the marginal household. In a general equilibrium context, there would be intra-marginal household at each location who might be willing to pay less to live in the formal sector and more in the slum. For large-scale conversions, this is a source of upward bias in our calculations.

7. CONCLUSIONS

This paper develops a framework for analysing a growing monocentric city in which residents make housing and location choices and developers sink capital in built structures, acting on the expectation of future rental income. We tailor the model to a developing country, so include an informal or slum sector in which structures are cheap and malleable, so capital is not sunk and construction can take place even if land-rights are insecure. We show how the model predicts the conversion of land from informal to formal as the city grows, and how areas go through successive waves of teardown and redevelopment. The model provides a platform for further work, such as exploring the role of expectations in urban growth, endogenizing urban productivity, and moving from a monocentric to polycentric urban structure.

The application is to Nairobi, a fast-growing city that, we argue, has many features in common with other developing cities. Based on a unique data set constructed from high-resolution aerial photographs, we study the city in cross-section (2015) and change (2003–15). We characterize how building height and ground cover, in both the formal and informal sectors, vary with distance from the city centre and use this cross-section to calibrate key parameters of the model. We find—consistent with assumptions in our model—that, closer to the centre, both sectors offer more built volume per unit area; the formal sector by building tall and the informal by crowding (i.e. more cover per unit area). We derive estimates of parameters of the two building technologies, indicating that land-rent (as opposed to construction) cost, is a much higher share of total rental income in the informal sector than in the formal. Turning to changes through time, the model does well in predicting the pattern of slum growth and two waves of formal sector development that occurred in the period 2003–15. One was redevelopment of formal structures relatively close to the city centre, with redevelopment typically trebling building height as predicted by the model. The other was new formal sector development further out, on the fringe between formal and informal areas.

A feature of Nairobi and other developing cities is the persistence of slums in high land value areas near the city centre. This is rationalized in the model as being due to a package of barriers to formalization. In Nairobi, many of these barriers arise as “slumlords”, who rent out property at market rates, do not have sound claims on the land, so resist a process of formalization that might deprive them of their flow of rental income. We use estimates from the model to calculate the losses due to these barriers, and find they are very substantial. Even if slumlords were to be paid-off (compensating for a loss of perpetual control), conversion would yield a surplus of about \$18,000 per slum household in a context where they pay about \$500–700 a year for their housing.

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Supplementary Data

Supplementary data are available at *Review of Economic Studies* online. And the replication packages are available at <https://dx.doi.org/10.5281/zenodo.3948449>.

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